REALIZATION OF EQUIVARIANT CHAIN COMPLEXES

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ABSTRACT. We discuss a question appearing in a recent article by A. Sikora [4] concerning the vanishing of certain differentials in the Leray-Serre spectral sequence for a Poincaré duality space equipped with a $\mathbb{Z}/p$-action.

Let p be an odd prime and let X be a finite dimensional connected $\mathbb{Z}/p$-CW complex which fulfills Poincaré duality over $\mathbb{F}_p$, the field with p elements. By definition, this means that $H^*(X;\mathbb{F}_p)$ is finitely generated over $\mathbb{F}_p$, there is a natural number $n \geq 0$ and an element $\nu \in H_n(X;\mathbb{F}_p)$ such that the map

$$H^i(X;\mathbb{F}_p) \to H_{n-i}(X;\mathbb{F}_p)\,,\, c \mapsto c \cap \nu\,,$$

is an isomorphism for all $i \in \mathbb{Z}$. Let $g \in \mathbb{Z}/p$ be a fixed generator. With the induced $\mathbb{Z}/p$-operation, $H^*(X;\mathbb{F}_p)$ is a graded $\mathbb{F}_p[\mathbb{Z}/p]$-module and as such it has a direct sum decomposition

$$H^*(X;\mathbb{F}_p) \cong V_1^* \oplus V_2^* \oplus \ldots \oplus V_p^*\,,$$

where each V_i^* is a free graded module over $\mathbb{F}_p[\xi]/(1-\xi)^i$ and multiplication by ξ corresponds to multiplication by g. In his recent article [4], A. Sikora discusses the following question:

Question (cf. [4], remarks following Theorem 1.4.) Let $V_2^* = V_3^* = \ldots = V_{p-1}^* = 0$ and let $\mathbb{Z}/p$ act on X with nonempty fixed point set. Consider the cohomological Leray-Serre spectral sequence with coefficients $\mathbb{F}_p$ for the Borel fibration

$$X \hookrightarrow E\mathbb{Z}/p \times_{\mathbb{Z}/p} X \to B\mathbb{Z}/p\,.$$

Do all the differentials $d_r : E_r^{i,j} \to E_r^{i+r,j-r+1}$ in this spectral sequence vanish, if $i \geq n$ and if r is odd and greater than 1?

We will show by an explicit example that this is false in general. In a first step, we construct our example on an algebraic level as a certain equivariant chain complex. In a second step, we realize this chain complex p-locally as

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the cellular chain complex of a $\mathbb{Z}/p$-CW complex which is thickened up and doubled in order to get a smooth $\mathbb{Z}/p$-manifold. The idea of this approach might be of independent interest for the construction of other equivariant spaces with prescribed homological properties.

Theorem 3 shows that a modified version of the above question can be answered affimatively. Related to this observation are the results in [1].

Consider the following chain complex C_* of $\mathbb{Z}[\mathbb{Z}/p]$-modules:

$$\mathbb{Z}[\mathbb{Z}/p] \stackrel{\tau}{\to} \mathbb{Z}[\mathbb{Z}/p] \stackrel{\nu}{\to} \mathbb{Z}[\mathbb{Z}/p] \stackrel{\tau\circ\tau}{\to} \mathbb{Z}[\mathbb{Z}/p] \stackrel{\nu}{\to} \mathbb{Z}[\mathbb{Z}/p] \stackrel{\tau}{\to} \mathbb{Z}[\mathbb{Z}/p] \to$$
$$0 \to 0 \to 0 \to 0 \to \mathbb{Z}\,.$$

The map ν is multiplication by $1 + g + \ldots + g^{p-1}$ and τ is multiplication by $1 - g$. We regard this chain complex as being graded over the natural numbers, the modules occuring above sitting in degrees $10, 9, \ldots, 0$, with differentials of degree -1 (and C_* being completed by zero modules in degrees larger than 10). Because $\tau \circ \nu = \nu \circ \tau = 0$, we see that C_* is indeed a chain complex. In a first step, we realize this complex as the cellular chain complex of a $\mathbb{Z}/p$-CW complex. As we are going to work with $\mathbb{F}_p$-coefficients later on, we consider the problem p-locally. We denote by $\mathbb{Z}_{(p)}$ the integers localized at p.

Proposition 1. *There is a 10-dimensional $\mathbb{Z}/p$-CW complex Y whose equivariant cellular chain complex with coefficients $\mathbb{Z}_{(p)}$ is $\mathbb{Z}_{(p)}[\mathbb{Z}/p]$-isomorphic to $C_* \otimes \mathbb{Z}_{(p)}$.*

Proof. The space Y is constructed inductively, starting with the one point union of p spheres of dimension 5 permuted cyclically by the action of $\mathbb{Z}/p$ (and with fixed common basepoint). Suppose that $5 \leq k \leq 9$ and that the k-skeleton $Y^{(k)}$ of Y has been constructed. We have to show that given an element $c \in H_k(Y^{(k)}; \mathbb{Z}_{(p)})$, there is a map $S^k \to Y^{(k)}$ which, in homology, maps the fundamental class of S^k to $\lambda \cdot c$, where λ is an integer not divisible by p. Then a bunch of free $\mathbb{Z}/p$-cells of dimension $k+1$ can be attached equivariantly to $Y^{(k)}$ according to the respective differential

$$C_{k+1} \otimes \mathbb{Z}_{(p)} \to C_k \otimes \mathbb{Z}_{(p)}$$

(up to a $\mathbb{Z}_{(p)}[\mathbb{Z}/p]$-linear automorphism of $C_{k+1} \otimes \mathbb{Z}_{(p)}$).

In order to achieve this aim, it is enough to show that the p-local Hurewicz map

$$\pi_k(Y^{(k)}) \otimes \mathbb{Z}_{(p)} \to H_k(Y^{(k)}; \mathbb{Z}_{(p)})$$

is surjective. For $k \leq 8$, we use the fact that in the Atiyah-Hirzebruch spectral sequence

$$E^2_{i,j} = \widetilde{H}_i(Y^{(k)}; \pi^s_j \otimes \mathbb{Z}_{(p)}) \Longrightarrow \pi^s_{i+j}(Y^{(k)}) \otimes \mathbb{Z}_{(p)}$$

converging to the p-local stable homotopy of $Y^{(k)}$, the terms $E^2_{i,j}$ vanish for $j = 1, 2$ as p is odd. Hence, all elements in $E^2_{i,0}$ with $i \leq 8$ are permanent cocycles (recall that $Y^{(k)}$ is 4-connected). Freudenthal's suspension theorem shows that the canonical map

$$\pi_k(Y^{(k)}) \rightarrow \pi^s_k(Y^{(k)})$$

is surjective. This map remains surjective after tensoring with $\mathbb{Z}_{(p)}$ and the desired surjectivity of the p-local Hurewicz map above is established.

If $p > 3$ and $k = 9$, the same argument completes the construction of Y, because $E^2_{i,j} = 0$ for $j = 3$ in this case (and Freudenthal's suspension theorem still gives a surjection from the unstable to the stable homotopy of $Y^{(9)}$ in degree 9). However, because $\pi^s_3 \cong \mathbb{Z}/24$, in the case that $p = 3$, we must show that $Y^{(9)}$ can be constructed in such a way that the fourth differential d^4 vanishes on $E^4_{9,0}(Y^{(9)})$.

The equivariant map $\sigma : Y^{(5)} = S^5 \vee S^5 \vee S^5 \rightarrow S^5$ which is the identity on each copy of S^5 (and with the trivial action on the target S^5) extends to an equivariant map

$$Y^{(6)} = (S^5 \vee S^5 \vee S^5) \cup_\phi (D^6 \cup D^6 \cup D^6) \rightarrow S^5$$

because $\sigma \circ \phi : S^5 \cup S^5 \cup S^5 \rightarrow S^5$ is homotopic to a constant map. We call this extended map σ, as well. The composition of σ with the inclusion $S^5 \rightarrow S^5 \vee S^5 \vee S^5 \subset Y^{(6)}$ of any S^5 summand is the identity. Because 3-locally the homotopy groups $\pi_6(S^5) = \pi_7(S^5) = 0$, one sees that σ extends to a 3-local equivariant map $\sigma : Y^{(8)} \rightarrow S^5$. We have $H_8(Y^{(8)}; \mathbb{Z}_{(3)}) \cong \mathbb{Z}_{(3)}$ and the E_∞-term of the Atiyah-Hirzebruch spectral sequence converging to the stable homotopy of $Y^{(8)}$ (always localized at 3) leads to a short exact sequence

$$0 \rightarrow \mathbb{Z}/3 \rightarrow \pi^s_8(Y^{(8)}) \rightarrow \mathbb{Z}_{(3)} \rightarrow 0\,.$$

Hence there is an isomorphism

$$f : \pi_8(Y^{(8)}) \cong \pi^s_8(Y^{(8)}) \cong \mathbb{Z}/3 \oplus \mathbb{Z}_{(3)}\,,$$

(the first isomorphism is the Freudenthal suspension theorem, again), but the choice of f is not canonical. Nevertheless, with respect to any such f, the $\mathbb{Z}_{(3)}$-summand is mapped isomorphically to $H_8(Y^{(8)}) \cong \mathbb{Z}_{(3)}$ under the Hurewicz map - this follows from the fact that the Hurewicz map is represented by an edge homomorphism in the Atiyah Hirzebruch spectral sequence.

Let $x = f^{-1}((0,1)) \in \pi_8(Y^{(8)})$ and $i : S^5 \rightarrow Y^{(5)} \hookrightarrow Y^{(8)}$ be chosen such that $\sigma \circ i$ is the identity. The image of $i_* : \pi_8(S^5) \rightarrow \pi_8(Y^{(8)})$ is in the kernel of the Hurewicz map, because $\operatorname{im} i \subset Y^{(5)}$. In particular, we find an element $c \in \pi_8(Y^{(8)})$ which is in the kernel of the Hurewicz map and such that $\sigma_*(c) = -\sigma_*(x) \in \pi_8(S^5)$. The element $x + c \in \pi_8(Y^{(8)})$

is then in the kernel of σ_* and is sent to a generator of $H_8(Y^{(8)})$ under the Hurewicz map. Thus, we can attach the first 9-cell to $Y^{(8)}$ in such a way that the composition of the attaching map with σ is null homotopic. By equivariance of σ, the composition with σ of the attaching maps of the other two 9-cells are null homotopic as well and σ extends (3-locally) to a map $Y^{(9)} \to S^5$ which factors the identity $S^5 \to S^5$. By naturality of the Atiyah-Hirzebruch spectral sequence, this shows that for this $Y^{(9)}$, we have indeed $d^4 = 0$ on $E^4_{9,0}$.

□

Note that $\mathbb{Z}/p$ acts on Y with exactly one fixed point. Let T be an oriented compact smooth $\mathbb{Z}/p$-manifold with boundary which is $\mathbb{Z}/p$-homotopy equivalent to Y (for the construction of such an equivariant smooth thickening, see, for example, [3] Theorem 2.4 and Remark 2.5 with $B = Y^{\mathbb{Z}/p}$ and U and E product bundles). Now define $X = T \cup_{\partial T} (-T)$ as the oriented double of T. The space X is an oriented closed smooth $\mathbb{Z}/p$-manifold and in particular satisfies Poincaré duality over $\mathbb{F}_p$. By use of Poincaré duality for T and excision, the long exact cohomology sequence of the pair (X, T) becomes

$$\ldots \to H^{i-1}(T;\mathbb{F}_p) \to H_{\dim T - i}(T;\mathbb{F}_p) \to H^i(X;\mathbb{F}_p) \to H^i(T;\mathbb{F}_p) \to \ldots$$

One sees (at least, if $\dim T \geq 22$ which we can assume) that $H^i(X;\mathbb{F}_p) \cong \mathbb{F}_p$, if $i = 0, 5, 7, 8, 10, n-10, n-8, n-7, n-5, n$, where $n = \dim X$, and $H^i(X;\mathbb{F}_p) = 0$ for all other values of i. In particular, the induced $\mathbb{Z}/p$-action on $H^*(X;\mathbb{F}_p)$ is trivial. Let $E_r^{*,*}$ be the spectral sequence for the Borel fibration $X \hookrightarrow E\mathbb{Z}/p \times_{\mathbb{Z}/p} X \to B\mathbb{Z}/p$ with coefficients in $\mathbb{F}_p$. The following theorem shows that X can be used in order to answer Sikora's question in the negative.

Theorem 2. *The third differential $d_3 : E_3^{i,*} \to E_3^{i+3,*-2}$ is different from zero for all even $i \geq 2$.*

Proof. Because T is an equivariant retract of X and T is $\mathbb{Z}/p$-homotopy equivalent to Y, we only need to show the latter statement for the spectral sequence of the Borel construction for Y, which we denote by the same symbol $E_r^{*,*}$. For $r \geq 2$, this is a bigraded module over

$$H^*(\mathbb{Z}/p;\mathbb{F}_p) \cong \mathbb{F}_p[t] \otimes \Lambda(s)$$

where s and t are considered as indeterminates of bidegree $(1,0)$ and $(2,0)$ respectively and where $\Lambda(s)$ is the exterior algebra on s. Furthermore, the differential on $E_r^{*,*}$ is $\mathbb{F}_p[t] \otimes \Lambda(s)$-linear. In the following, we abbreviate $E\mathbb{Z}/p \times_{\mathbb{Z}/p} Y$ by $Y_{\mathbb{Z}/p}$ and take coefficients in $\mathbb{F}_p$, throughout. By use of the localization theorem, we have

$$H^*(Y_{\mathbb{Z}/p})[t^{-1}] \cong H^*(Y^{\mathbb{Z}/p} \times_{\mathbb{Z}/p} E\mathbb{Z}/p)[t^{-1}] \cong \mathbb{F}_p[t, t^{-1}] \otimes \Lambda(s) ,$$

because we have exactly one fixed point. It is now convenient to localize the Leray-Serre spectral sequence right away: For $r \geq 2$, we set $\overline{E}_r^{*,*} = E_r^{*,*}[t^{-1}]$ and denote the induced differential on this localized spectral sequence (living in the first two quadrants) by $\overline{d}_r$.

By induction on $r \geq 2$, it is not difficult to show that the map

$$E_r^{i,j} \to E_r^{i+2,j}$$

given by multiplication with t, is a surjection, if $0 \leq i < r-1$, and an isomorphism, if $i \geq r-1$. In particular, the canonical map

$$E_3^{i,j} \to \overline{E}_3^{i,j}$$

is an isomorphism, if $i \geq 2$. Hence, it suffices to show that $\overline{d}_3^{i,*} \neq 0$ for all even i.

Recall that by construction of the Leray Serre spectral sequence, there is a decreasing filtration

$$\ldots \supset \mathcal{F}_{\gamma-1}H^*(Y_{\mathbb{Z}/p}) \supset \mathcal{F}_\gamma H^*(Y_{\mathbb{Z}/p}) \supset \mathcal{F}_{\gamma+1}H^*(Y_{\mathbb{Z}/p}) \supset \ldots$$

such that

$$E_\infty^{i,j} \cong \mathcal{F}_i H^{i+j}(Y_{\mathbb{Z}/p}) / \mathcal{F}_{i+1}H^{i+j}(Y_{\mathbb{Z}/p}) .$$

As in [2], we now define an induced filtration on the localized module $H^*(Y_{\mathbb{Z}/p})[t^{-1}]$ as follows:

$$x \in \mathcal{F}_\gamma \left(H^*(Y_{\mathbb{Z}/p})[t^{-1}] \right) \Leftrightarrow t^c \cdot x \in \mathcal{F}_{\gamma+2c} H^{*+2c}(Y_{\mathbb{Z}/p}) \text{ for } c \gg 0 .$$

This makes sense, because, using the remarks from the preceding paragraph, multiplication by t induces isomorphisms

$$\mathcal{F}_\gamma H^*(Y_{\mathbb{Z}/p}) \cong \mathcal{F}_{\gamma+2} H^{*+2}(Y_{\mathbb{Z}/p})$$

if $\gamma \geq 11$, because $E_{12}^{*,*} = E_\infty^{*,*}$ for dimension reasons. It follows that

$$\overline{E}_\infty^{i,j} \cong \mathcal{F}_i \left(H^{i+j}(Y_{\mathbb{Z}/p})[t^{-1}] \right) / \mathcal{F}_{i+1} \left(H^{i+j}(Y_{\mathbb{Z}/p})[t^{-1}] \right) .$$

Hence, because $H^*(Y_{\mathbb{Z}/p})[t^{-1}]$ is a module of rank 2 over the graded field $\mathbb{F}_p[t, t^{-1}]$, the same must be true for $\overline{E}_\infty^{*,*}$. We will show that this cannot hold, if $\overline{d}_3^{i,*} = 0$ for some even i.

In [2], Theorem 1, we constructed operators $\Gamma_{1,r} : E_r^{*,*} \to E_r^{*,*+1}$ for $r \geq 1$ that, for $r = 1$, can be identified with the Bockstein operator β on $H^*(Y; \mathbb{F}_p)$ associated to the short exact coefficient sequence

$$0 \to \mathbb{Z}/p \to \mathbb{Z}/p^2 \to \mathbb{Z}/p \to 0 .$$

Because these operators $\Gamma_{1,r}$ act as derivations, they are $\mathbb{F}_p[t]$-linear (for $r \geq 2$) and induce corresponding operators on $\overline{E}_r^{*,*}$, $r \geq 2$. We also cite the fact that these operators commute with $\overline{d}_r$ up to sign. Notice that

$\beta(H^i(Y;\mathbb{F}_p)) = 0$ for $i \neq 7$ and $\beta : H^7(Y;\mathbb{F}_p) \to H^8(Y;\mathbb{F}_p)$ is an isomorphism (this explains our choice of the chain complex C_*). Using the operator $\Gamma_{1,2}$, we get $\overline{d}_2 = 0$ by a simple diagram chase and therefore

$$\overline{E}_3^{i,j} \cong H^j(Y;\mathbb{F}_p) \otimes (\mathbb{F}_p[t,t^{-1}] \otimes \Lambda(s))^i \,.$$

We now assume that $\overline{d}_3^{i,*} = 0$ for some even i. We know that $\overline{E}_3^{*,*}$ is two-periodic in the horizontal direction and so our assumption implies that $\overline{d}_3^{i,*}$ vanishes for all even i. Because $\overline{d}_3$ has odd horizontal degree, commutes with s up to sign and $s \cdot s = 0$, the preceding isomorphism shows that $\overline{d}_3^{i,*} = 0$ for all odd i, as well. Hence we get $\overline{d}_3 = 0$. For $j = 0, 5, 7$ the differential $\overline{d}_4^{*,j}$ vanishes for dimension reasons. But also $\overline{d}_4^{*,8}$ vanishes by commutativity of the diagram

$$\begin{array}{ccc} \overline{E}_4^{*,7} & \xrightarrow{\overline{d}_4} & \overline{E}_4^{*+4,4} = 0 \\ \Gamma_{1,4}\downarrow & & \Gamma_{1,4}\downarrow \\ \overline{E}_4^{*,8} & \xrightarrow{-\overline{d}_4} & \overline{E}_4^{*+4,5} \end{array}$$

where the first vertical arrow is an isomorphism. In a similar way, the differential $\overline{d}_4^{*,10}$ is zero.

For dimension reasons, we have $\overline{d}_5 = 0$. Because the $\mathbb{Z}/p$-operation on Y has a fixed point, the projection map $Y_{\mathbb{Z}/p} \to B\mathbb{Z}/p$ has a section and thus factors the identity $B\mathbb{Z}/p \to B\mathbb{Z}/p$. By comparing the localized spectral sequences for $Y_{\mathbb{Z}/p}$ and $*_{\mathbb{Z}/p} = B\mathbb{Z}/p$, this implies that the differential $\overline{d}_r : \overline{E}_r^{*,r-1} \to \overline{E}_r^{*+r,0}$ vanishes for all r. Altogether, we get

$$\dim_{\mathbb{F}_p[t,t^{-1}]} \overline{E}_\infty^{*,*} \geq 6$$

because the only possibly nonzero differential is $\overline{d}_6 : \overline{E}_6^{*,10} \to \overline{E}_6^{*+6,5}$. However, this contradicts the above calculation of this dimension. $\square$

Theorem 3. *Let X be a $\mathbb{Z}/p$-CW complex with finitely generated cohomology over $\mathbb{F}_p$ in every degree. Assume that in the decomposition of $H^*(X;\mathbb{F}_p)$ described at the beginning of this note, we have $V^*_{p-1} = 0$ and that $H^*(X;\mathbb{Z}_{(p)})$ does not contain $\mathbb{Z}/p$ as a direct summand. Then, in the localized spectral sequence $\overline{E}_r^{*,*}$ with $\mathbb{F}_p$-coefficients for the Borel construction of X, the differential $\overline{d}_r$ vanishes, if r is odd and $r > 1$.*

Proof. The Bockstein operator β is zero on $H^*(X;\mathbb{F}_p)$, therefore, by [2] Proposition 9 (which holds for any $\mathbb{Z}/p$-CW complex with vanishing Bockstein), each $\overline{E}_r^{*,*}$ is free over $\mathbb{F}_p[t,t^{-1}] \otimes \Lambda(s)$. The universal coefficient sequence

$$0 \to H^*(X;\mathbb{Z}_{(p)}) \otimes \mathbb{Z}/p^2 \to H^*(X;\mathbb{Z}/p^2) \to H^{*+1}(X;\mathbb{Z}_{(p)}) * \mathbb{Z}/p^2 \to 0$$

shows that $H^*(X;\mathbb{Z}/p^2)$ is free over $\mathbb{Z}/p^2$: Because $\mathbb{Z}/p$ is not a direct summand of $H^*(X;\mathbb{Z}_{(p)})$, the modules on the left and on the right are free over $\mathbb{Z}/p^2$. By the vanishing of the Bockstein operator, the short exact coefficient sequence

$$0 \to \mathbb{Z}/p \to \mathbb{Z}/p^2 \to \mathbb{Z}/p \to 0$$

induces a short exact sequence

$$0 \to H^*(X;\mathbb{F}_p) \to H^*(X;\mathbb{Z}/p^2) \to H^*(X;\mathbb{F}_p) \to 0\,.$$

The image of the second map consists of elements that are divisible by p as $H^*(X;\mathbb{Z}/p^2)$ is free over $\mathbb{Z}/p^2$. Consequently, we get an induced isomorphism $H^*(X;\mathbb{Z}/p^2) \otimes \mathbb{F}_p \cong H^*(X;\mathbb{F}_p)$. Proposition 6 in [2] (or a little representation theory) now implies that $V_2^* = \ldots = V_{p-2}^* = 0$ in the decomposition of $H^*(X;\mathbb{F}_p)$. Together with the assumption $V_{p-1}^* = 0$, this shows that we can take $\overline{E}_r^{0,*}$ as an $\mathbb{F}_p[t,t^{-1}] \otimes \Lambda(s)$-basis of the free module $\overline{E}_r^{*,*}$: We have

$$\overline{E}_2^{*,j} \cong H^*(\mathbb{Z}/p; H^j(X;\mathbb{F}_p))[t^{-1}] \cong V_1^j \otimes (\mathbb{F}_p[t,t^{-1}] \otimes \Lambda(s))^*\,,$$

so no basis element can sit on an odd column for $r \geq 2$. We now assume that there is an odd r so that $\overline{d}_r \neq 0$. The facts that $\overline{d}_r$ is $\Lambda(s)$-linear, that $\overline{E}_r$ is free over $\Lambda(s)$ with basis elements located on even columns, that the horizontal part of the bidegree of $\overline{d}_r$ is odd and that $s \cdot s = 0$ imply that still $\overline{d}_r^{i,j} = 0$, if i is odd. Hence, $\overline{d}_r^{0,*} \neq 0$ and not all basis elements in $\overline{E}_r^{0,*}$ survive to $\overline{E}_{r+1}^{0,*}$. Therefore, multiplication by s cannot be surjective as a map $\overline{E}_{r+1}^{0,*} \to \overline{E}_{r+1}^{1,*}$ and the bigraded module $\overline{E}_{r+1}^{*,*}$ is not free over $\Lambda(s)$, contrary to what we said before. □

REFERENCES

[1] Ch. Allday, B. Hanke and V. Puppe, *Poincaré duality in P.A. Smith theory*, Proc. AMS **131** (2003), 3275-3283.

[2] B. Hanke, *Inner products and $\mathbb{Z}/p$-actions on Poincaré duality spaces*, Forum Math. **15** (3) (2003), 439-454.

[3] K. Pawałowski, *Fixed point sets of smooth group actions on disks and Euclidean spaces*, Topology **28** (1989), 273-289.

[4] A. Sikora, *Torus and $\mathbb{Z}/p$ actions on manifolds*, Topology (to appear).

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