European Parliament

A mathematical formula for determining the EP composition

KEY FINDINGS

- This paper presents a technique, called FPS, to obtain methods to allocate the number of seats of each Member State in the EP, while respecting all the requirements of Article 14(2) TEU.

- All obtained methods give **degressive proportionality before rounding**. To determine a FPS method, it is necessary to choose the value of **two parameters**: the **percentage F** of seats *distributed equally among all Member States*, and the **percentage P** of seats *distributed in proportion to the populations* of the Member States.

- The method with F=10 and P=50 is used to obtain the allotment for the parliamentary term 2024-2029 and to compare it with the **allocation agreed by the EP and the Council** in June 2023, giving similar results. After that, **the results using the same FPS method** for the election held at the years: 2019, 2014, 2009 and 2004 are graphically compared with the allocation agreed for these elections (Annex I). In all cases, **both allocations are close**.

- **Our recommendation** is to use this method to determine the composition of the European Parliament. But **one can choose the values of F and P**. It would also be possible to use another FPS method with parameters **close to F=10 and P=50**, because then one would get **distributions similar** to those the EP and the Council have negotiated in the last two decades. The method with P=8, F=46 is one of the most interesting ones for Member States with intermediate population; therefore, this method is also recommended.

- Another recommendation is that **the size of the EP could be set automatically**, so that the limit 96 **does not imply an additional degressivity** to the most populous Member State. In this case, the size of the EP for the legislative term 2024-2029 would have been 707 seats, when using the FPS method cited above.

Introduction

In June 2024 a new election will be held in the European Union (EU) in order to obtain representation for its 27 Member States in the European Parliament (EP). Lisbon Treaty establishes a minimum of 6 seats for each Member State and a maximum of 96 seats. The total number of seats cannot exceed 751, and the allotment to the states has to be degressively proportional before rounding (Annex II). The above restrictions are set out in Article 14(2) TEU.

EN

There's no permanent formula to determine the allocation of EP seats per Member State, yet. In September 2023, the EP and Council agreed on a composition of the EP with 720 seats for the June 2024 election. To this aim, the MSs' population number of January 2022 were taken into account. According to the population number used, the agreed distribution of seats meets the requirements of the Lisbon Treaty.

However, there is a large number of seat distribution techniques or formulas that equally comply with the Lisbon Treaty restrictions without changing the size of 720, while respecting the 6 and 96 seats limitation. This makes it difficult to choose the ideal distribution formula.

However, in degressivity it would be reasonable to request that when comparing the proportion of their populations/seats for three consecutive countries in population, these population costs per seat change in a similar way as their populations change. This is not the case for many of the distributions that satisfy Degressive Proportionality (DP).

A mathematical formula can simplify the distribution of seats between EU countries and even give fairer results than those obtained through negotiations in terms of population costs per seat.

This is achieved by the method proposed in this work and also by many other different formulas; some of them are in "Compilation: Two briefing and one in-depth analysis" provided for the AFCO Committee[1].

The FPS technique to get degressive proportionality

We will describe a technique that allows to obtain adjusted quotas that verify DP (Annex III). It is applicable for any size h of EP that is established, for any number of Members States n and whatever their populations, p_i, of the states. For the 2024 election it has been established $h = 720$ seats, there will be $n = 27$ Member States and the populations taken into account are those shown in Table 1, which correspond to those used by the EP and the Council during the negotiations in 2023. But all these data could be different and the FPS technique would be applied in the same way.

The idea is very simple: the fraction of seats f_i, before rounding and imposing limitations to be allocated to the state i, is obtained as the sum of three values.

Firstly, a percentage of EP seats is distributed equally among the n states. For example, 10% of the seats could be distributed equally (**Fixed**). With this we are already choosing one of the two parameters that will lead to a particular FPS method. Our first parameter has been: $F = 10$. If the size of the EP is 720, 10% means that 72 seats are distributed equally among the 27 Member States. Each Member State receives the following seats:

$$\frac{0.10 * 720}{27} = \frac{72}{27} \cong 2.67$$

Next, another percentage of seats is distributed in proportion to the populations. We will note this parameter with the letter P (**Proportional**). For example, half of the seats, which would be 360 seats, are distributed proportionally to the populations, so it has been chosen as the second parameter: $P = 50$. Thus, as the total population of the EU is 447 533 143 inhabitants, Ireland with a population of 5 060 004 receives the following seats:

$$\frac{0.50 * 720 * 5\,060\,004}{447\,533\,143} = \frac{1\,821\,601\,440}{447\,533\,143} \cong 4.07,$$

---

[1] The composition for the European Parliament. Committee on Constitutional Affairs. Directorate General for Internal Policies of the Union. Policy Department for Citizen's Rights and Constitutional Affairs. PE 583.117 - February 2017

while Portugal with slightly more than twice the population of Ireland, exactly 10 352 042 inhabitants, receives 8.33 seats in this distribution, which is slightly more than double as Ireland.

Only 40% of the seats remain to be distributed: $S = 100 - (F + P) = 100 - 60 = 40$. These seats are distributed in proportion to the **square roots of the** populations, an easy operation to do with any elementary calculator.

Ireland would be allocated the following number of seats in this third allocation (the denominator, 91 209, is the sum of the square roots of the populations):

$$\frac{0.40 * 720 * \sqrt{5\,060\,004}}{91\,209} = \frac{288 * 2\,249.45}{91\,209} = \frac{647\,842}{91\,209} \cong 7.10$$

This is far more than Ireland received in the proportional distribution, although fewer seats are distributed this time.

However, Portugal now receives 10.16 seats which is nowhere near twice as much as Ireland. This is because the square root increases more slowly as populations grow, and so it is appropriate to use it to achieve degressive proportionality. The square roots of the populations of these two states are: 2249 and 3217, so the second quantity is nowhere near twice as large as the first.

Therefore, the adjusted quotas for Ireland and Portugal are

$$Adjusted(Ireland) \cong 2.67 + 4.07 + 7.10 = 13.84$$

$$Adjusted(Portugal) \cong 2.67 + 8.33 + 10.16 = 21.16$$

Therefore, if rounding is done to the nearest integer, Ireland expects to receive 14 seats with the FPS method we are using, and Portugal 21.

However, before rounding, the effect of the minimum 6 and the maximum 96 must be reflected. As seen in Table 1, Germany contributes 2.4 seats left over from its adjusted quota, to remain at 96. Malta and Luxembourg, they only need 0.64+0.28=0.92 to reach 6 seats. The difference is 1.48 seats, which is equivalent to a very small increase in the adjusted quotas of the other 24 countries. Those of Ireland and Portugal would become 13.87 and 21.21, with which the rounding to the nearest integer becomes 14 and 21 again. Other Member States benefit from the limitations; for example, the adjusted quotas of France and Poland are 83.25 and 52.33 however they receive 84 and 53 seats respectively.

From now on, we refer to the FPS method as the one used with $F = 10,$ and $P = 50$. Therefore, when we use a different FPS method we have to indicate its F and P values. For example FPS(8-46), when 8% of the seats are distributed equally, 46% are distributed in proportion to populations and the other 46% in proportion to the square root of populations.

Allocation agreed and FPS allocation for 2024-2029 parliamentary term

Table 1 shows (column 2) the seats agreed by the EP and the Council, to each state, in the term 2024-2029. It also shows all the calculations of the FPS method. It can be seen, in the last column, that **with FPS no state would have lost more than one seat** with respect to the agreed distribution.

Table 1. Allocations agreed and FPS for 720 seats. Legislative term 2024-2029.

State	Seats 2024	Populat. 2022	$\sqrt{Pop.}$	Fixed	Prop.	Squar.	Adjusted Quotas	Seats FPS	Diff
Germany	96	83 203 320	9 122	2.67	66.93	28.80	98.40	96	
France	81	67 842 582	8 237	2.67	54.57	26.01	83.25	84	+3
Italy	76	59 607 184	7 721	2.67	47.95	24.38	74.99	75	-1
Spain	61	47 432 805	6 887	2.67	38.15	21.75	62.57	63	+2
Poland	53	37 654 247	6 136	2.67	30.29	19.38	52.33	53	
Romania	33	19 038 098	4 363	2.67	15.31	13.78	31.76	32	-1
Netherlands	31	17 734 036	4 211	2.67	14.27	13.30	30.23	30	-1
Belgium	22	11 631 136	3 410	2.67	9.36	10.77	22.79	23	+1
Greece	21	10 603 810	3 256	2.67	8.53	10.28	21.48	22	+1
Czechia	21	10 545 457	3 247	2.67	8.48	10.25	21.40	22	+1
Sweden	21	10 440 000	3 231	2.67	8.40	10.20	21.27	21	
Portugal	21	10 352 042	3 217	2.67	8.33	10.16	21.15	21	
Hungary	21	9 689 010	3 113	2.67	7.79	9.83	20.29	20	-1
Austria	20	8 967 500	2 995	2.67	7.21	9.46	19.34	19	-1
Bulgaria	17	6 838 937	2 615	2.67	5.50	8.26	16.42	16	-1
Denmark	15	5 864 667	2 422	2.67	4.72	7.65	15.03	15	
Finland	15	5 541 241	2 354	2.67	4.46	7.43	14.56	15	
Slovakia	15	5 434 712	2 331	2.67	4.37	7.36	14.40	14	-1
Ireland	14	5 060 004	2 249	2.67	4.07	7.10	13.84	14	
Croatia	12	3 862 305	1 965	2.67	3.11	6.21	11.98	12	
Lithuania	11	2 805 998	1 657	2.67	2.26	5.29	10.21	10	-1
Slovenia	9	2 107 180	1 452	2.67	1.69	4.58	8.95	9	
Latvia	9	1 875 757	1 370	2.67	1.51	4.32	8.50	9	
Estonia	7	1 331 796	1 154	2.67	1.07	3.64	7.38	7	
Cyprus	6	904 700	951	2.67	0.73	3.00	6.40	6	
Luxembourg	6	643 648	802	2.67	0.52	2.53	5.72	6	
Malta	6	520 971	722	2.67	0.42	2.28	5,36	6	
Total	720	447 533 143	91 209	72	360	288	720	720	

The graph below reflects the apportionment agreed by the EP and the Council in September 2023 (the red dots) and the function used by the FPS method to obtain the adjusted quotas (the green line).
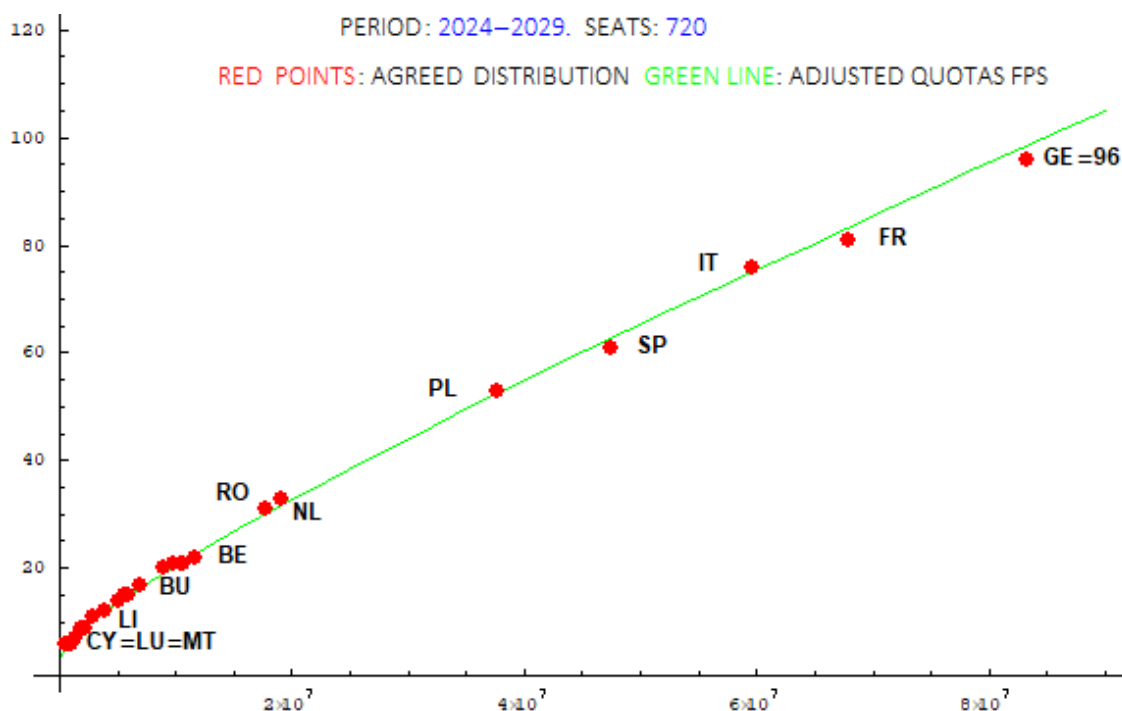
If a red dot is below the curve, it means that FPS would have allocated more seats to it, except in the case of Germany because it is affected by the 96-seat ceiling. For example, the dots for France, Spain, Belgium, etc. are below the curve, which is why the FPS method allocates more seats to them than they will have in the 2024 elections.

If a red dot is above the curve, it means that FPS would have allocated fewer seats to that country. There is no point far above the curve and therefore **no country would have lost more than one seat under the FPS method compared to** the agreed distribution.

In general, the **agreed distribution** for the 2024 elections **differs little** from that which would have resulted **from the FPS** method, not more than one seats except for France and Spain.

DP for the agreed allocation and DP before rounding for FPS allocation

The third column of Table 2 contains the DP for the current distribution, i.e. the ratio between population and seats for each EU country, and the fourth column contains the DP before rounding for the FPS distribution. In this case, the denominators are not exactly the values in the Adjusted Quotas column of Table 1, because it has been necessary to include the minimum and maximum constraints of 6 and 96 seats (this affects Malta, Luxembourg and Germany).

Comparing three consecutive values in the second column and in the fourth column we see that the middle quantity is similar in both cases. For example, France, Italy and Spain are three consecutive countries in the population size and we see that the population of Italy is much closer to that of France than that of Spain (differences are 9 and 12 million inhabitants, respectively) and the population cost per seat of Italy is also much closer to France than that of Spain (differences are 20 000 and 36 000, respectively). This is the case with FPS method, because as can be seen in the graph above, if we look at the part of the graph that corresponds to three consecutive populations it looks similar to a straight line.

Table 2. Seats for 2024, populations in 2022 and seat costs

State	Population 2022	Population/ Seats 2024	Population/ Quotas-Lim.
Germany	83 203 320	866 701	866 701
France	67 842 582	837 563	812 975
Italy	59 607 184	784 305	792 907
Spain	47 432 805	777 587	756 256
Poland	37 654 247	710 457	717 785
Romania	19 038 098	576 912	598 015
Netherlands	17 734 036	572 066	585 232
Belgium	11 631 136	528 688	509 090
Greece	10 603 810	504 943	492 496
Czechia	10 545 457	502 165	491 508
Sweden	10 440 000	497 143	489 710
Portugal	10 352 042	492 954	488 197
Hungary	9 689 010	461 381	466 388
Austria	8 967 500	448 375	462 654
Bulgaria	6 838 937	402 290	415 354
Denmark	5 864 667	390 978	389 227
Finland	5 541 241	369 416	379 737
Slovakia	5 434 712	362 314	376 510
Ireland	5 060 004	361 429	364 728
Croatia	3 862 305	321 859	321 641
Lithuania	2 805 998	255 091	274 079
Slovenia	2 107 180	234 131	234 993
Latvia	1 875 757	208 496	220 140
Estonia	1 331 796	190 257	179 976
Cyprus	904 700	150 783	141 066
Luxembourg	643 648	107 275	107 275
Malta	520 971	86 829	86 829
Total	447 533 143		

Negotiations for the composition of the EP seats can be very difficult, because each state tends to prefer as many representatives as possible and the inequalities for the DP to be verified sometimes leave room for various options.

For example, Poland could have requested that in 2024 the EP should have 728 seats and that the 8 new seats should be allocated to Poland, because it would obtain the same number of seats than Spain (but not exceed) and the population cost per seat would be greater than that of Romania.

$$C_{Poland(61)} = \frac{37\ 654\ 247}{61} = 617\ 283 > 576\ 912 = C_{Romania}$$

However, such an allocation is not feasible with an FPS method like the one used above, because the population cost of Poland's seats (before rounding) will be much closer to the population cost of Spain's seats than to the population cost of Romania's seats. This is because the population of Poland is much closer to that of Spain than to that of Romania (population differences are 10 and 18 million respectively).

Similarly, other countries could claim more seats in a negotiation on the basis of compliance with the DP.

Other FPS methods: What values to use for F and P?

It is in the interest of sparsely populated states that F be very large and P very small. More populous countries are interested in just the opposite. Countries with intermediate populations, say those that currently have a population between 5 and 20 million, are interested in both percentages being small and almost all seats being distributed in proportion to the square root of the populations.

The choice of the two parameters of an FPS method is therefore a political decision. Perhaps there are two groups of States with different interests, the more populated ones preferring a large P and those with an intermediate population who prefer it to be large S. The less populous states are protected by the minimum of 6 seats.

Mathematics could provide an answer by calculating the percentages that produce a distribution as close as possible to the one agreed for 2024. But they would be somewhat different from the values obtained if we use the distribution agreed with 2017 populations for the composition after Brexit, or the one agreed for 2014 with 2012 populations, etc.

Analyzing many agreed distributions from the 2004 EP election to the 2024 one, it would lead us to suggest that F can be between 8 and 10 and P between 45 and 50.

Thus, a much more interesting FPS method for countries with an intermediate population would be the one obtained with $F = 8, P = 46$ which would mean that $S = 46$. How much would the representation of the countries that would benefit most from such a change have increased by 2024? **One seat only**. Six countries with populations between the population of Slovenia and Romania would benefit by one seat (Annex IV, rounding Webster). The countries that would lose representation would be Latvia, among the least populated, and the most populated countries from Poland onwards.

Therefore, I believe that the political decision turns out to be much easier than it seemed. If a similar degressivity is desired as in the past, with the FPS technique, it is not necessary to make a major change in the two parameters with respect to the values used in the example, $F = 10, P = 50$. In Annex I we show graphically the behaviour of the FPS method in all the parliamentary terms from 2004 to 2019.

Method for rounding, and entry of new states

We have seen that we can either benefit less populous states by increasing the seats affecting the first parameter, F, or benefit the more populous countries by increasing the percentage of the second

parameter, P, or benefit those in the middle zone by increasing the seats that are distributed in proportion to the square root of the populations.

Therefore, it is of no interest to look for a method for rounding fractions that benefits one or the other again. **Therefore, the proposal we make is to use Webster's (or odd divisors) method, as it is an unbiased, consistent and monotonic method.** Other rounding methods are also possible, such as d'Hondt, which favour the most populous states.

In practice, it is very simple to obtain a distribution with Webster's method from the adjusted quotas, it would be enough to divide them by the numbers 1, 3, 5, ..., etc. and keep the largest quotients to assign to each state as many seats as the largest quotients have been obtained with their adjusted quota, but always at least 6 and no more than 96. It is described in Annex V.

Entry of new Member States in the course of a legislature

An FPS method is applicable to determine the number of seats to be allocated to a state joining the EU over the course of a legislature.

It is sufficient to calculate its adjusted quota using the same expression that was used for the other states. Thus, if a state were to join in the parliamentary term 2024-2029 and we had used the method described in section 2, the adjusted quota of a new state, with a number of inhabitants in 2022 equal to Pop.2022, would be:

$$Adjusted(Pop.2022) = \frac{72}{27} + \frac{0.50 * 720 * Pop.2022}{447\ 533\ 143} + \frac{0.40 * 720 * \sqrt{Pop.2022}}{91\ 209}$$

To assign the representation of a new state, it would be sufficient to enter in the above expression its population as of the date of calculation of the quota adjustment function, i.e. its population in 2022, and round the result to the nearest integer.

Table 3. Number of seats for new states

State	Pop. 2022	Adjusted(Pop.2022)	Seats
Albania	2793592	10.19	10
Bosnia-Herz	3460000	11.33	11
Macedonia N.	1837114	8.42	8
Moldova	2603729	9.86	10
Montenegro	617683	5.65	6
Serbia	6797105	16.37	16
Turkey	84680273	99.84	96
Ukraine	40997698	55.86	56

If any of the countries applying for EU membership were to be accepted before 2029, according to the populations in 2022, their representatives until May 2029 would be those listed in the last column of Table 3.

It is possible that the additions carried out during a legislative term will produce a size of the EP exceeding 751 seats. For example, if the Balkan countries (Albania, Bosnia-Herzegovina, Macedonia, Montenegro and

Serbia) were to join the EU in 2028, the EP would have 771 members until the end of the 2029 legislative term.

Non-discriminatory EP Size

The maximum limitation, 96, can be a penalty for the most populous countries. This penalty occurs when a country's adjusted quota is much higher than 96. For the upcoming legislative term, with 720 seats and without the maximum limitation, Germany should have received 98 seats under the FPS method, because its adjusted quota is somewhat higher than 98, so Germany would have a penalty of 2 seats with the 96 limitation. It is unreasonable for one country to have a penalty that does not affect the others.

To avoid such a penalty, the size of the EP can be adjusted.

As a possible criterion, the size of the EP could be set at the maximum size for which the most populous country is not penalised by the 96 limit.

For the next election that size would have been 707 seats instead of 720. Because when the maximum limit of 96 seats is not considered, the table below contains the allocation that Germany would receive for different sizes of PE.

EP Size	702	703	704	705	706	**707**	**708**
Germany seats	95	96	96	96	96	**96**	**97**

Therefore, the size 707 is the maximum with which Germany receives 96 seats, unaffected by the limitation when PFS is used.

The graph below shows the seats (blue dots) that each Member State would have received by 2024 with 707 seats using FPS.
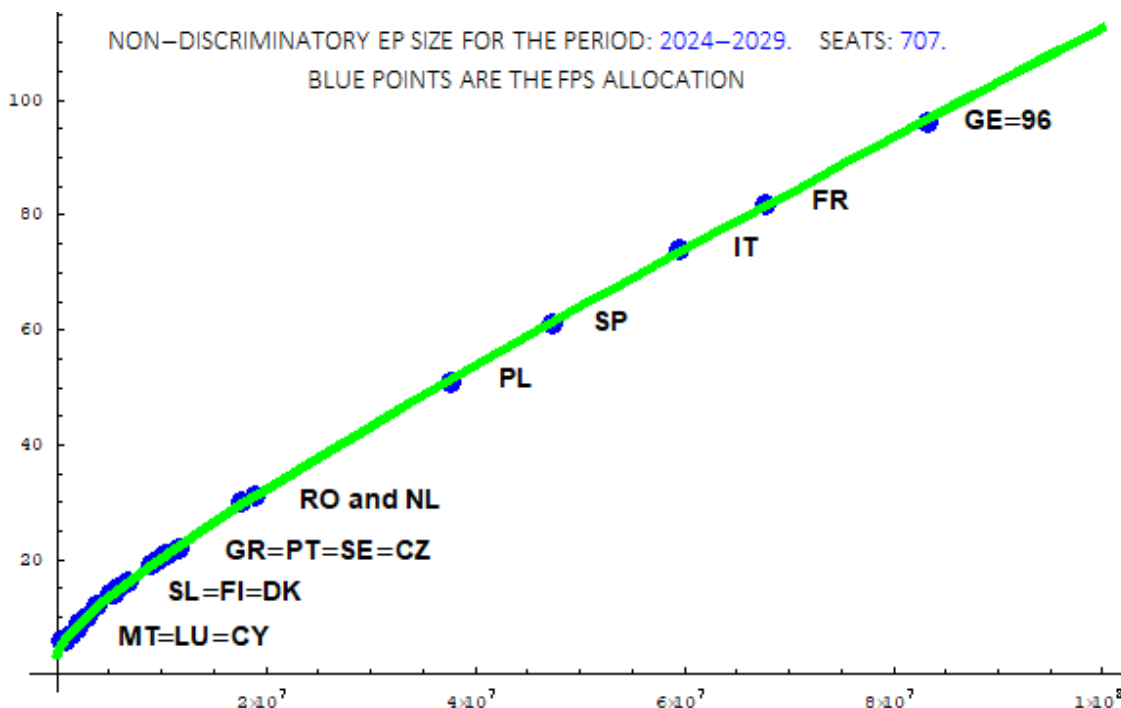
Recommendations

Our recommendation is to use the FPS method to distribute EP seats among Member States. The values of F and P values can be changed to others, in order to favor either the most populous states, or the least populous ones, or those with an intermediate population. This is a political decision.
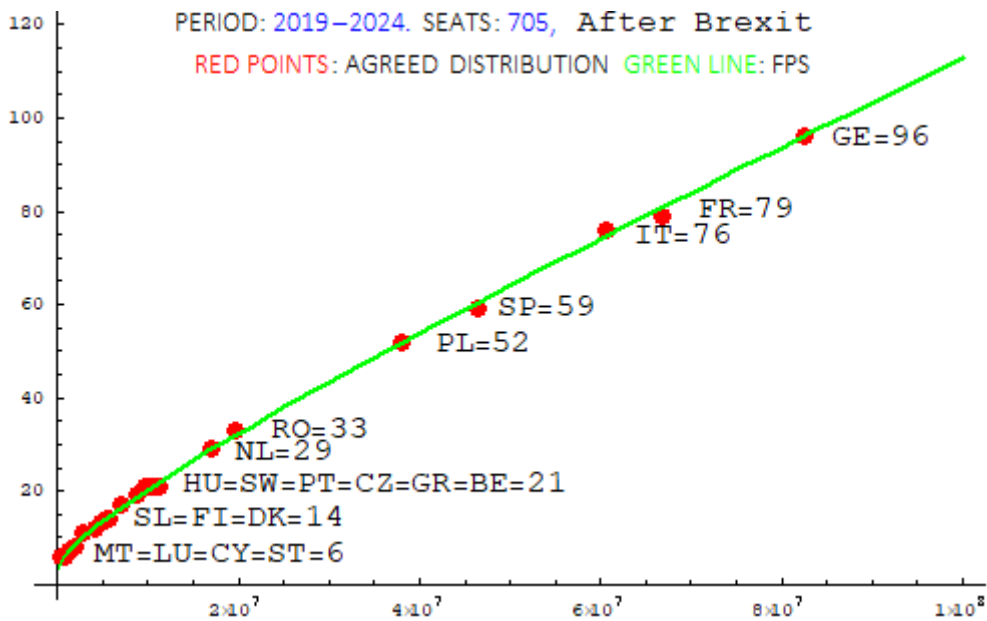
This recommendation can be justified on the basis that every FPS method checks DP before rounding, it is applicable for changes in the number of states, their populations and the size of the EP. It allows to calculate the allocation of a state joining the EU over a parliamentary term of time. In addition, the suggested parameter values $F = 10, P = 50$ would have given allocations from 2004 to 2024 close to those agreed through negotiations, and parameter values close to them, produce allocations that differ very little from these standard values.

We also recommend linking the size of the EP to the FPS technique so that the maximum possible EP size is chosen for which the most populous EU country does not receive a penalty due to the 96 limit.
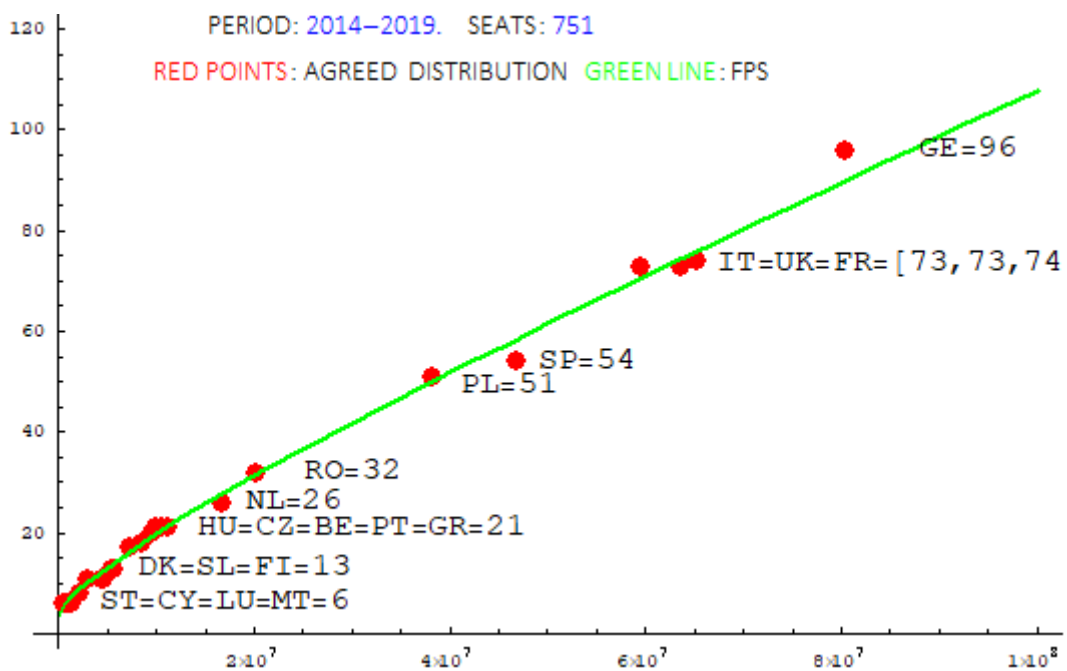
Annex I. Comparison of agreed allocations from 2004 to 2019 parliamentary terms with the FPS ones

The following shows, graphically, all seat distributions, from the parliamentary term beginning in 2004 to the parliamentary term beginning in 2019, with the quota fitting curve used the FPS method with $F = 10, P = 50$. The agreed distributions are the red dots. The adjusted quotas would be on the green line, on the vertical of the red dot.
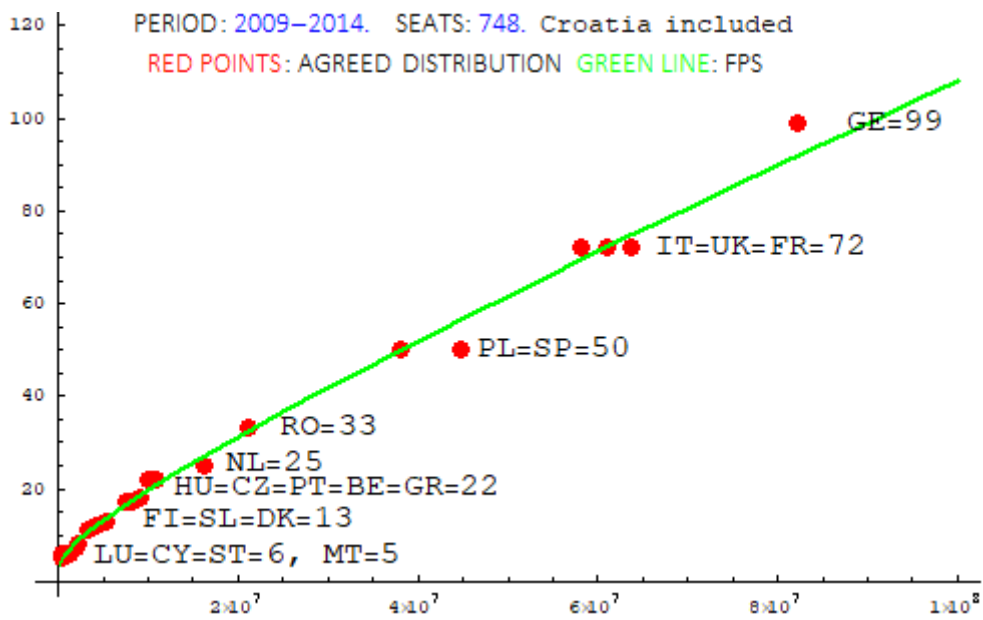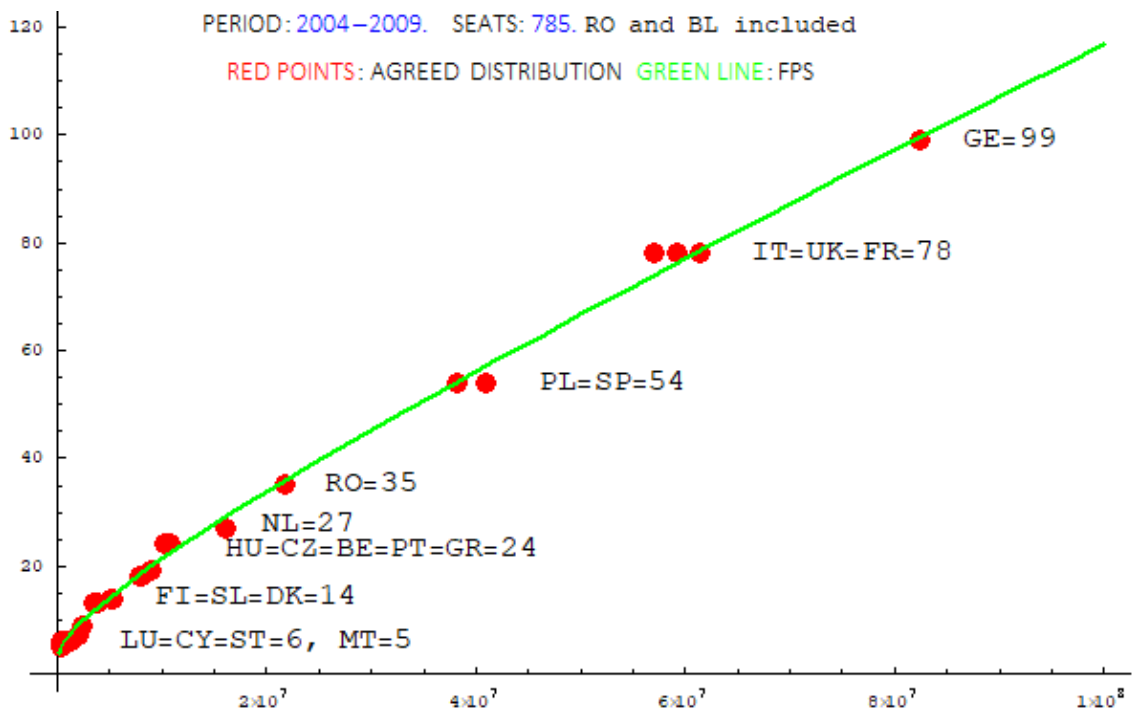
Populations in 2017. Seats in 2020

Populations in 2012. Seats in 2014 (Croatia is included)

Populations in 2007. Seats in 2009

PERIOD: 2009–2014. SEATS: 748. Croatia included

RED POINTS: AGREED DISTRIBUTION GREEN LINE: FPS

GE=99

IT=UK=FR=72

PL=SP=50

RO=33

NL=25

HU=CZ=PT=BE=GR=22

FI=SL=DK=13

LU=CY=ST=6, MT=5

Populations in 2002. Seats in 2004

PERIOD: 2004–2009. SEATS: 785. RO and BL included

RED POINTS: AGREED DISTRIBUTION GREEN LINE: FPS

GE=99

IT=UK=FR=78

PL=SP=54

RO=35

NL=27

HU=CZ=BE=PT=GR=24

FI=SL=DK=14

LU=CY=ST=6, MT=5

Annex II. The DP before rounding

The idea of DP means that the more populous states will be under-represented compared to the least populous. A concept that was ambiguously established at the beginning of this century and that was qualified in the Cambridge Compromise report prepared in 2011, at the request of the Member European Parliament (MEP) Andrew Duff, in the sense that decreasing proportionality has to be verified before rounding.

To understand DP before rounding, we consider the number h of seats in the EP, the number n of Member States in the EU and let $p_1, p_2, \dots, p_n$ be their most recent populations,

$$p_1 \geq p_2 \geq \ \dots \ \geq p_n.$$

In the 2024-2029 parliamentary term, $h = 720$ seats, $n = 27$ Member States, and the populations $p_1, p_2, \dots, p_n$ correspond to Germany, France, ..., Malta (respectively).

A mathematical formula gives the state i its adjusted quota, f_i which represents the fraction of seats to which it is entitled. The fractions of seats f_i add up to h and follow the same order as the populations. The most populous country, currently Germany, should have the largest fraction of seats, followed by the fraction of the second most populous country, and so on down to the country with the smallest number of inhabitants, which will have the smallest fraction. In other words,

$$f_1 \geq f_2 \geq \ \dots \ \geq f_n$$

The formula must ensure that the adjusted quotas verify degressive proportionality. This means that, for any two states $i < j$, one has the following inequality

$$\frac{p_i}{f_i} \geq \frac{p_j}{f_j}$$

Or written in extended form, for the current 27 states, it would be:

$$\frac{p_1}{f_1} \geq \frac{p_2}{f_2} \geq \frac{p_3}{f_3} \geq \frac{p_4}{f_4} \geq \ \dots \ \geq \frac{p_{25}}{f_{25}} \geq \frac{p_{26}}{f_{26}} \geq \frac{p_{27}}{f_{27}}$$

If the fractions f_i were integer numbers and corresponded to the seats allocated to each state, the above inequalities would indicate that each German seat represents more Germans than each French seat represents Frenchmen, and so on down to Malta, which would be the country for which the seats have the smallest cost. But the above inequalities have to be rounded to integer numbers, because they are fractions, and so we say decreasing proportionality before rounding.

Any method can be used for rounding to integer numbers, e.g. Webster's method which rounds to the nearest integer.

When rounding to integers, it is possible that quotas may need to be adjusted because the most populous country (or even one of the next most populous countries) has more than 96 seats, which would leave its allocation at 96; or, conversely, the least populous country (or even one of the next most populous countries) may have a rounding of less than 6 seats, in which case it must be raised to 6, to ensure the minimum and maximum requirements.

In practice, the process of rounding fractions with minimum and maximum requirements to integer numbers is a very simple problem, because it is solved in proportional distribution problems, and that is

what is done when rounding fractions of seats to integer numbers. What is not immediate is to obtain the fractions with decreasing proportionality.

The roundings obtained are the seats $s_1, s_2, \ldots, s_n$ which each of the n states and will form a sequence with the same order as the populations and the fractions, i.e:

$$s_1 \geq s_2 \geq \quad \ldots \geq s_n$$

With the allocations s_i we cannot be sure that the ratios between populations and seats in each country follow a degressive proportionality. If $i < j$ we cannot be sure that

$$\frac{p_i}{s_i} \geq \frac{p_j}{s_j}$$

On other occasions it is possible to find a distribution that verifies DP after rounding, but it may require forcing the same allocation of seats to many Member States, causing an injustice between the most populous and the least populous of that group.

Annex III. The quota adjustment function always verifies DP

The function $A(.)$ that results for an FPS method is of the following type

$$A(x) = a + bx + c\sqrt{x}$$

where the coefficients are obtained from the number of seats to be distributed, the number of seats to be distributed h, the number n of states, their populations p_i and the values of F y P of the method. Specifically, their values are:

$$a = \frac{F * h}{100 * n}, \qquad b = \frac{P * h}{100 * \sum_{i=1}^{n} p_i}, \qquad c = \frac{S * h}{100 * \sum_{i=1}^{n} \sqrt{p_i}}$$

None of them is negative. The function $A(.)$ is going to be evaluated on populations and is therefore an increasing function. So a state with a higher population than another will have a higher adjusted quota and is guaranteed a higher fraction of seats. Therefore, it does not receive fewer seats.

For the degressivity of the adjusted quotas we have to analyse the behaviour of the ratios between the population x and it adjusted quota $A(x)$ for $x > 0$:

$$C(x) = \frac{x}{A(x)} = \frac{1}{A(x)/x} = \frac{1}{\frac{a}{x} + b + \frac{c}{\sqrt{x}}}$$

These ratios increase as the population increases. Therefore, the adjusted quotas verify DP. The same is true for fractions obtained by multiplying the adjusted quota s by a positive number.

Moreover, if positive fractions

$$f_1 \geq f_2 \geq \quad \dots \quad \geq f_n$$

verify DP and two constant values, m y M, are intermediate between f_1 y f_n; for example m is between f_{n-3} and f_{n-2}, and M is between f_2 y f_3, that is:

$$f_1 \geq f_2 \geq \quad \dots \quad f_{n-2} \geq m \geq f_{n-1} \geq f_n$$

$$f_1 \geq f_2 \geq M \geq f_3 \geq \quad \dots \geq f_{n-1} \geq f_n$$

Then the sequence

$$f_1, f_2, \quad \dots, \quad f_{n-2}, m, m$$

$$M, M, f_3, \quad \dots, f_{n-1}, f_n$$

satisfy DP. This results in the readjustment of quotas so that the roundings add up to the number of seats to be distributed and the minimum and maximum limits are met, $m = 6$ and $M = 96$. Therefore, every FPS method satisfy DP before rounding.

Annex IV. Comparison - Allocations using: Two FPS methods (10-50 and 8-46) with two rounding (Webster and d'Hondt)

State	Popul. 2022	Agreed Seats	FPS$_{1W}$ 10-50	FPS$_{1d'H}$ 10-50	FPS$_{2W}$ 8-46	FPS$_{2d'H}$ 8-46
Germany	83 203 320	96	96	96	96	96
France	67 842 582	81	84	85	82	83
Italy	59 607 184	76	75	76	74	75
Spain	47 432 805	61	63	64	62	63
Poland	37 654 247	53	53	53	52	53
Romania	19 038 098	33	32	32	32	32
Netherlands	17 734 036	31	30	30	31	31
Belgium	11 631 136	22	23	23	23	23
Greece	10 603 810	21	22	22	22	22
Czechia	10 545 457	21	22	21	22	22
Sweden	10 440 000	21	21	21	22	21
Portugal	10 352 042	21	21	21	21	21
Hungary	9 689 010	21	20	20	21	20
Austria	8 967 500	20	19	19	20	20
Bulgaria	6 838 937	17	16	16	17	16
Denmark	5 864 667	15	15	15	15	15
Finland	5 541 241	15	15	14	15	15
Slovakia	5 434 712	15	14	14	15	14
Ireland	5 060 004	14	14	14	14	14
Croatia	3 862 305	12	12	12	12	12
Lithuania	2 805 998	11	10	10	10	10
Slovenia	2 107 180	9	9	9	9	9
Latvia	1 875 757	9	9	8	8	8
Estonia	1 331 796	7	7	7	7	7
Cyprus	904 700	6	6	6	6	6
Luxembourg	643 648	6	6	6	6	6
Malta	520 971	6	6	6	6	6
Total	447 533 143	720	720	720	720	720

Agreed Seats: EP composition during 2024-2029 parliamentary term.

FPS$_{1W}$: FPS method with F=10 and P=50, rounding with Webster (standard FPS method)

FPS$_{1d'H}$: FPS method with F=10 and P=50, rounding with d'Hondt.

FPS$_{2W}$: FPS method with F=8 and P=46, rounding with Webster.

FPS$_{2d'H}$: FPS method with F=8 and P=46, rounding with d'Hondt.

Green files: same number of seats on the four FPS allotments.

Annex V. Implementation of the FPS technique using a spreadsheet

by *Antonio Palomares Bautista* (member of the research team led by V. Ramírez González, at the University of Granada)

In the following lines, we describe the functionality of the sheet, the position of the data, and how the allotment is performed. With this tool, users can adjust the method variables, view the calculations performed, and obtain the allotment. Furthermore, all the formulas used can be viewed and checked.

The spreadsheet is largely self-explanatory because of the column headers. All the data and the calculations are on the same sheet. Broadly, at the top left you can modify the values of F, P (and consequently get S), below are the populations, and the adjusted quotas are calculated. On the right is the table of divisors to make the distribution with the Webster method (Sainte-Laguë). Further below, there is a graph displaying the results.

The spreadsheet can be found in .ods and .xls formats on the page

www.ugr.es/local/anpalom/FPS.html .

The data that can be modified (problem variables) are the following: Populations (B9:B35), parliament size (B3), percentages F (B4) and P (B5).

The sheet calculates the value of the S percentage (B6), the square roots of the populations (D9:D35), the adjusted quotas for each state (E9:E35) and the distribution of seats that the FPS method assigns to each state (F9:F35).

The sheet is prepared for 27 member states, a minimum of 6 seats and a maximum of 96.

Usually spreadsheet programs have the Automatic Recalculation option activated, and thus any change in the problem variables will automatically generate a new distribution. If the Automatic Recalculation option is not activated, the sheet can be recalculated with Ctrl+Shift+F9 in LibreOffice (or Shift+F9 in Excel).

The distribution requires applying the Webster (Sainte-Laguë) method, which is a method of divisors or a method of highest averages, with a minimum of 6 and a maximum of 96. The next lines explain how the distribution is technically done, although this explanation is not necessary to use the spreadsheet.

The distribution is carried out on the right side of the sheet. The Webster (Sainte-Laguë) method can be applied, as it would be done on a sheet of paper, by dividing the adjusted quotas of each state by the odd numbers (J7:DB7), and in the resulting table of quotients, we choose the larger ratios. Each odd number corresponds to a number of seats to be assigned, these seats are in J8:DB8. Due to the minimum and maximum size of each state, it is only necessary to divide by the odd numbers corresponding to seats 7 to 96, but for completeness seats 1 to 97 are shown.

As each state receives at least 6 seats, they are allocated due to this minimum 6*27=162 (G2) seats, and must be distributed using Webster (Sainte-Laguë) the total number of seats (e.g. 720) minus 162. This number is located on G3. In this way, if a total of 720 seats are distributed, it is required to distribute using the table 720-162=558 seats. This distribution is done by locating the G3 largest quotients in the table. Fortunately, spreadsheet programs have a function to find the k-th largest number in a range of data, which is LARGE (in Spanish the function is more descriptively called K.ESIMO.MAYOR). So in the G4 position, we can find the key number of the allotment that is the G3-th largest value of the quotients table P9:DA35.

Only one last step is needed to obtain the allotment. For each Member State, we have to count the number of quotients in the table greater than or equal to G4. This number of seats, plus the six of the minimum

allotment is the final allotment of the state, which can be found in the table at positions F9:F35 under the header 'Allocation FPS'.

It may be interesting to have marked the quotients that give rise to the allocation of seats. This marking may consist of changing the color or typography of the quotients. Unfortunately this mark is not recalculated automatically, but must be done manually by applying a conditional format to the data range P9:DA35 (or modifying the existing one), marking the values in the range that are greater than or equal to G4.

Just below the state populations, an example of a simple graph is included, but it is modifiable. The included example shows the results of the FPS apportionment along with those of the 2024 agreement. Each distribution is shown with a square or rhombus. The populations are measured on the horizontal axis and the seats assigned in the distributions are measured on the vertical axis.