Degressive proportionality in the European Union

KEY FINDINGS

- Allocating seats in the **European Parliament** (EP) according to a selected **mathematical formula** based on the populations of the Member States, allows us to avoid potential problems which may occur with any change in the number of Member States or with any considerable variation of their population.
- There exists a plethora of mathematical systems for the seats apportionment that agree with the bounds adopted by the Treaties and the rule of **degressive proportionality**. One of the simplest is the **base + prop** scheme, known also as the **Cambridge Compromise**.
- The **Modified Cambridge Compromise** (**base + power** scheme) is better suited in the case of the predicted exit of the United Kingdom from the EU than the original Cambridge Compromise, and results in the **minimum transfer of seats in the EP**, regardless of the size of the EP, with the rounding method adjusted to the size.
- Brexit provides a unique opportunity to implement a **smooth transition** to a **new balanced allocation system** in such a way that each Member State obtains at least the current number of seats in the EP. Such solutions exist also for an appropriately reduced size of the Parliament.
- The **minimum size** of the EP for which such a smooth solution exists in case of the **Modified Cambridge Compromise** is **721** (according to the current population data).
- Transition to one of the systems mentioned above will increase the share of representatives for a few of the largest Member States, and will reduce it for the medium-sized ones. Thus, to preserve the overall **balance of power** in the **European Union**, one should consider a simultaneous modification of the voting system in the **Council of the European Union**. For this purpose we recommend the **degressive proportional** system called the **Jagiellonian Compromise** that strengthens the voting power of the medium-sized states.

1. ALLOCATING SEATS IN THE EUROPEAN PARLIAMENT

According to the Treaty on European Union (EU) (in particular Article 14(2))[1] and the Council of the European Union Decision of 28 June 2013 establishing the composition of the European Parliament (EP)[2], the apportionment of seats in the EP **should be based on the principle of degressive proportionality** further technically specified in Article 1 of the Decision as follows:

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- *the allocation of seats in the European Parliament **shall fully utilise the minimum and maximum numbers set** by the Treaty on European Union in order to reflect as closely as possible the sizes of the respective populations of Member States,*
- *the ratio between the population and the number of seats of each Member State **before rounding to whole numbers** shall vary in relation to their respective populations in such a way that each Member of the European Parliament from a more populous Member State represents more citizens than each Member from a less populous Member State and, conversely, that the larger the population of a Member State, the greater its entitlement to a large number of seats*[3].

Since we have analysed this principle thoroughly in the paper entitled «Mathematical aspects of degressive proportionality» published four years ago[4], here we shall present only a brief résumé and refer the reader to the paper itself and the references therein for further details.

Degressive proportionality

The notion of **degressive proportionality**[5] plays a crucial role in the current apportionment scheme for the European Parliament. The principle of degressive proportionality enshrined in the Lisbon Treaty was probably borrowed from discussions on taxation rules, where the term appeared as early as the nineteenth century, when many countries introduced income tax for the first time in their history. It was already included in the debate on apportionment in the Parliament in the late 1980s[6], but at first it was a rather vague idea that gradually evolved into a formal legal (and mathematical) term in the Lamassoure & Severin report[7] adopted by the European Parliament in 2007.

The notion hardly appears in the constitutional solutions of the apportionment problem adopted either inside or outside the EU, where the proportional apportionment schemes seem to be prevalent. However, one can find several cases in political practice where degressively proportional solutions have been implemented, though not necessarily precisely defined and not necessarily under this name.

Firstly, many allocation systems that reserve a minimum number of seats in a political body, for all subunits represented, usually fail to be proportional, and so, some amount of degressive proportionality seems to be a natural solution in this case. The most famous example that comes to mind here is the Electoral College that formally elects the President and Vice President of the United States of America, where each state is allocated as many electors as it has Senators (equal base) and Representatives (proportional representation, with at least one seat per state) in the United States Congress. The idea of combining these two approaches to the apportionment problem was first put forward by one of the Founding Fathers of the United States and future American President, James Madison[8] in 1788.

Secondly, we can find at least two examples from European political practice: the apportionment of seats both in the upper house of the German Parliament[9] (*Bundesrat*), and in the electoral body comprising the members of the twelve Provincial Councils (*Provinciale Staten*) that elects the Senate (*Eerste Kamer*) of the Dutch Parliament, that are also *de facto* though not *de jure* degressively proportional.

Thirdly, the distribution of votes in the Council of the European Union from the very beginning of the European Communities until quite recent times, when the system of 'double majority' was introduced, has reflected the principle of degressive proportionality[10].

There have also been suggestions in academic literature to apply this general principle to the apportionment process in some other parliamentary or quasi-parliamentary bodies, such as the projected Parliamentary Assembly of the United Nations[11].

Degressively proportional apportionment – an algorithm

There is a fundamental difference between **proportional** and **degressive proportional** apportionment. While the former is a precisely defined mathematical concept, where only the rounding procedure gives us some freedom of manoeuvre, the latter does not provide us with a single solution, but instead offers an infinite number of options from which to choose.

How to cope with such a plethora of options in a systematic way? One of the major mathematical approaches to the problem of degressively proportional apportionment in the European Parliament can be described by the following general scheme:

1. One has to choose a concrete characterisation of the size of a given Member State *i* by a number p_i (for example, equal to the total number of its inhabitants, citizens or voters), which we call here **population**, and precisely define by which means the required data should be collected and how often it should be updated. Then, one needs to transform these numbers by an **allocation function *A*** belonging to a given family (**allocation scheme**) indexed by some parameter ***d***, whose range of variability is determined by the requirement that the function fulfils constraints imposed by the Treaties: is **non-decreasing** and **degressively proportional**.
2. Additionally, the allocation function *A* has to satisfy certain **boundary conditions**: $A(p) = m$ and $A(P) = M$, where the population of the smallest and the largest state equals, respectively, *p* and *P*, with the smallest and the largest number of seats predetermined as, respectively, *m* and *M*. (In the case of the EP these quantities are explicitly determined by the Treaty and the Decision: ***m* = 6** and ***M* = 96**.)
3. To assign integer number of seats for each Member State one has to round the values of the allocation function, e.g., using one of three standard **rounding methods** (*upward*, *downward* or *to the nearest integer*).
4. Finally, one has to choose the **parameter *d*** in such a way that the sum of the seat numbers of all Member States equals the projected **number of seats in the EP** (***S***), solving (if possible) in *d* the equation:

$$\sum_{i=1}^{N} [A_d(p_i)] = S\ ,$$

where *N* stands for the number of Member States, p_i for the population of the *i*-th state ($i = 1, \ldots, N$), and [·] denotes the rounded number.

Thus to prepare a degressive proportional apportionment of seats for the EP we have to set **three variables**:

a. the **number of seats in the EP** – ***S***;
b. the **allocation scheme** – ***A***;
c. the **rounding method** – **[·]**.

Knowing a), b) and c), we can choose the appropriate parameter *d* and, consequently, a concrete allocation function resulting in the distribution of seats in the EP associated with the given variables. Though usually there is a whole interval of parameters satisfying this requirement, nonetheless, in a generic case, the distribution of seats established in this way is unique. This technique bears a resemblance to divisor methods in the proportional apportionment problem used first by Thomas Jefferson[12] in 1792.

Please note that in the Lamassoure and Severin definition of degressive proportionality it was postulated that this property holds for the number of seats after rounding the values of the allocation function to whole numbers. However, one can show that there exist such distributions of population that there is no solution to the apportionment problem satisfying such defined degressive proportionality[13]. Consequently, Grimmett *et al.* recommended[14] to weaken this condition and to amend the definition of degressive proportionality assuming that the property holds for the number of seats before rounding. Their proposal has been approved by the Constitutional Affairs Committee of the European Parliament (AFCO) and finally contained in Article 1 of the Decision.

Proposed forms of allocating schemes

In our papers published several years ago[15] we gathered together and analysed seven natural allocation schemes, i.e., seven one-parameter families of allocation functions[16], and studied their properties with the implementation to the apportionment for the EP under three rounding procedures (downward, to the nearest integer, upward):

- **base + prop**,
- **piecewise linear**,
- **quadratic (parabolic)**,
- **base + power**,
- **homographic**,
- **linear + hyperbolic**,
- **min-max proportional.**

All seven families mentioned above share a common element: the **linear** (**affine**) **allocation function**. This is undoubtedly the simplest allocation function one can imagine. However, under present circumstances, it would lead to a smaller parliament than the current one, but its size can serve as an indicator to estimate how many seats we can allocate freely besides the linear (or, more precisely, affine) distribution.

Note that all these solutions have been already discussed in the academic literature. The base + prop scheme, which seems to be the simplest of all these methods, was first analysed by Pukelsheim[17] and became the basis for the proposal, called 'Cambridge Compromise', elaborated in January 2011 by a group of mathematicians and political scientists[18], and discussed later by the Committee on Constitutional Affairs (AFCO) of the EP[19]. The piecewise linear scheme was proposed for the first time by the authors of this briefing[20] and, independently, by Ramírez González *et al.*[21] under the name of the Linear Spline Method. On the other hand, the quadratic (parabolic) scheme was advocated by Ramírez González and his co-workers in a series of papers[22]. The base + power scheme has been studied by many authors from Ramírez González *et al.*[23] to Grimmett *et al*.[24], although it can be traced to the paper of Theil and Schrage[25] from 1977. Note that a similar method was proposed for solving the taxation problem as early as the nineteenth century by the Dutch economist Cohen-Stuart[26]. The homographic scheme functions introduced by the authors[27], were also studied under the name of projective quotas by Serafini[28]. The linear + hyperbolic scheme was used both in the apportionment problem for the EP[29], as well as in the tax schedule proposed by the Swedish economist Cassel at the beginning of the twentieth century[30]. Finally, the proportional apportionment method with minimum and maximum requirements was considered by Balinski and Young[31]. Moreover, the linear allocation function was studied under the name of base + strict prop by Kellerman[32].

We have observed that all these solutions are quite similar (with the notable exception of *min-max proportional*), which is a consequence of the fact that our choice is limited by two factors: the predetermined shape of the graph of an allocation function, and the fact that the vast majority of seats are, in a sense, distributed in advance. However, one can observe that the results for the *parabolic*, *base + power*, and *homographic* allocation schemes lead to quite similar apportionments, whereas the choice of the *base + prop* scheme is advantageous for large countries, and the *piecewise linear* and *linear + hyperbolic* schemes seems to be beneficial for small countries[33].

In 2011 the authors of this briefing joined the group of mathematicians and political scientists endorsing the so-called '**Cambridge Compromise**'[34]. This allocation system, equivalent to the base + prop method with rounding to the nearest integer, was selected mainly because of its obvious simplicity[35]. However, this solution has been criticised for being 'not degressively proportional enough' and departing too much from the *status quo* by Moberg[36]. In 2012 a solution very similar to the base + power scheme was considered by Grimmett *et al.* as a step along a continuous transition from the negotiated *status quo* composition to the constitutionally principled Cambridge Compromise[37]. The crucial point in these discussions seems to be the meaning of the term 'degressive proportionality'. Is it only a less perfect form of (pure) proportionality, as it was actually suggested by some authors or is it a separate notion that requires distinct (and new) mathematical and political solutions, as Moberg claims? Personally, we incline towards the latter suggestion.

2. RECOMMENDED SOLUTIONS FOR THE EUROPEAN PARLIAMENT

As 'it is unclear whether the UK's 73 seats will be lost or reallocated'[38], we have analysed six possible choices for the size of the EP:

- 751 - with the UK;
- 751 - without the UK;
- 678 = 751 – 73 - without the UK;
- Optimal size - without the UK;
- Minimum size - with the UK;
- Minimum size - without the UK,

along with seven allocation schemes and three rounding methods, which overall results in **seventy seven different allocations** overall. Analysing all these solutions, we primarily take into account Article 4 of the Decision that requires *establishing a system which in future will make it possible, before each fresh election to the European Parliament, to allocate the seats between Member States in an **objective**, **fair**, **durable** and **transparent** way, **translating the principle of degressive proportionality** as laid down in Article 1, taking account of any change in their number and demographic trends in their population, as duly ascertained thus **respecting the overall balance of the institutional system** as laid down in the Treaties*[39].

As all analysed schemes 'translate the (mathematical) principle of degressive proportionality' into the political realm, and their mathematical form guarantees that the resulted apportionment would be indeed objective, fair, durable and transparent, we have looked for the solutions that **change the *status quo* as little as possible**[40] trying (in order):

- to minimize the number of seats transferred;
- to minimize the number of Member States loosing seats;
- to maximize the number of Member States gaining seats.

Such solutions would lead to a relatively **smooth transition** from the current apportionment into a new one. We call them **balanced solutions**. Note that **some transfer of seats is inevitable** in this case, firstly, because of demographic changes[41] and, secondly, since the present apportionment in the EP is, in a sense, erratic and irregular as a result of some historical bargaining[42], rather than objective considerations. To be more specific, one can say, with some degree of unavoidable inaccuracy, that there are three groups of Member States for which the result of projected changes will be, relatively or absolutely (depending on the future size of the EP)

- positive: France, the United Kingdom (if applicable), Spain, Estonia;
- neutral: Germany, Italy, Poland, the Netherlands, Denmark, Finland, Slovakia, Ireland, Croatia, Slovenia, Latvia, Cyprus, Luxembourg, Malta;
- negative: Romania, Belgium, Greece, the Czech Republic, Portugal, Sweden, Hungary, Austria, Bulgaria, Lithuania.

The results of our considerations are outlined below, divided according to two main criteria - the size of the EP and the presence of the British MEPs (see Tab. 1 and Tab. 2 for details):

EP 751 (with the UK)

We are still convinced that the 'Cambridge Compromise' (i.e. the **base + prop scheme with the rounding to the nearest integer**) gives here the simplest acceptable solution.

However, a balanced solution in this case is given by the **base + power scheme with the rounding downwards**.

EP 751 (without the UK)

Assume that the size of the EP remains unchanged after Brexit and all British seats are distributed among other Member States. Then the 'Cambridge Compromise' produces a solution with large (possibly too large) transfer of seats, especially to a few large states, while the balanced solution is given by the **base + power scheme rounding upwards**.

EP 678 (without the UK)

Assume that the size of the EP is reduced by the number of British seats. Here again the 'Cambridge Compromise' produces a solution with a large transfer of seats, while the balanced solution is given by the **base + power scheme with rounding downwards**.

EP Optimal size (without the UK)

Assume that the size of the EP is reduced by a smaller number than the number of British seats. We have been looking for the smallest size of the Parliament with no Member State losing seats. Several options are possible here with the balanced solution given in this case by the **base + power scheme with rounding to the nearest integer** and the size of the EP equal to **721**.

EP Minimum size (with or without the UK)

Assume that the simplest allocation function is chosen, i.e., **linear with the rounding to the nearest integer**. The resulting size of the EP would be either 718 (with the UK) or 640 (without the UK). Although probably politically hard to implement, this solution shows (approximately) how many seats can be in fact freely allocated in both situations: only 751 – 718 = 33 or 678 – 640 = 38, respectively.

An additional argument for the base + power scheme

The base + power scheme has an additional property called **super-proportionality**[43]. To illustrate this property consider two pairs of Member States: Romania/France and Belgium/Poland, with the similar population quotient (approx. 29.7%), and another such configuration: Sweden/Romania and Finland/Greece (approx. 50.6%). Note that in all these cases the seat quotient must be larger than population quotient because of degressive proportionality. However, in both cases (assuming the balanced solution and the current size of the EP: 28 states and 751 seats) the seat quotient is greater for the 'smaller' pair than for the 'larger' one, as we get 31.25% for Romania/France whereas 40% for Belgium/Poland, as well as 60% for Sweden/Romania whereas 68.42% for Finland/Greece. In other words, a super-proportional method leads to the following property of an allocation system (before rounding): The smaller a pair of states is, the larger the gain in seats of the smaller member in the pair over the larger one. Hence, if an allocation function is super-proportional, then the degressive proportionality acts more strongly for smaller states, and so such functions are, in a sense, more degressively proportional than others. Thus, this is in fact a kind of **degressive–degressive proportionality**.

Final recommendation for the European Parliament apportionment

*We recommend the adoption of the **Modified Cambridge Compromise**, i.e., **base + power system** as the solution that **minimizes the transfer of seats** and, at the same time, fulfilling all constitutional requirements and expressing more accurately **the principle of degressive proportionality** than other solutions considered. The **specific form of rounding** in the system should depend on the projective size of the EP and concrete population data, and should be chosen to further minimize the transfer of seats. Having at* *our disposal two extreme solutions: preserving the current size of the Parliament (751) or reducing the size by all British seats (to 678), we advocate, however, for an **intermediate solution**. Assuming that no Member State should lose any seat during the transition procedure, and simultaneously trying to minimize the size of the Parliament, we arrive at the '**optimal**' number of **721** representatives.*

Table 1: Alternative proposals for the EP apportionment depending on the EP size (with the UK). Here (CC) stands for the *base* + *prop* scheme, (MCC) for the *base* + *power* scheme, (L) for the *linear* allocation, (d) and (n) for the rounding, resp., *downwards* and *to the nearest integer*. *Total transfer of seats* is the sum of losses and gains in the number of seats. The *exponents* are computed for the *base* + *power* schemes.

Member State	Population	*Status quo*	EP+UK 751 (CC) (n)	EP+UK 751 (MCC) (d)	EP+UK Minimum (L) (n)
Germany	82 064 489	*96*	96	**96**	96
France	66 661 621	*74*	84	**80**	79
United Kingdom	65 341 183	*73*	83	**79**	78
Italy	61 302 519	*73*	78	**74**	73
Spain	46 438 422	*54*	61	**59**	57
Poland	37 967 209	*51*	50	**50**	47
Romania	19 759 968	*32*	29	**30**	27
Netherlands	17 235 349	*26*	26	**27**	25
Belgium	11 289 853	*21*	19	**20**	18
Greece	10 793 526	*21*	18	**19**	17
Czech Republic	10 445 783	*21*	18	**19**	17
Portugal	10 341 330	*21*	18	**19**	17
Sweden	9 998 000	*20*	17	**18**	17
Hungary	9 830 485	*21*	17	**18**	16
Austria	8 711 500	*18*	16	**17**	15
Bulgaria	7 153 784	*17*	14	**15**	13
Denmark	5 700 917	*13*	12	**13**	12
Finland	5 465 408	*13*	12	**13**	12
Slovakia	5 407 910	*13*	12	**12**	11
Ireland	4 664 156	*11*	11	**12**	11
Croatia	4 190 669	*11*	10	**11**	10
Lithuania	2 888 558	*11*	9	**9**	9
Slovenia	2 064 188	*8*	8	**8**	8
Latvia	1 968 957	*8*	8	**8**	8
Estonia	1 315 944	*6*	7	**7**	7
Cyprus	848 319	*6*	6	**6**	6
Luxembourg	576 249	*6*	6	**6**	6
Malta	434 403	*6*	6	**6**	6
UE-27	510 860 699	*751*	751	**751**	718
Total transfer of seats		0	66	42	56
Exponent		-	-	0.903	1

Population data based on the Council Decision 2016/2353 of 8 December 2016 amending the Council's Rules of Procedure. See the endnotes 16 & 19 for the formulae used to compute apportionments.

Table 2: Alternative proposals for the EP apportionment depending on the EP size (without the UK). Here (MCC) stands for the *base + power* scheme, (L) for the *linear* allocation, and (d), (n), and (u) for the rounding, resp., *downwards*, *to the nearest integer*, and *upwards*. *Total transfer of seats* is the sum of losses and gains in the number of seats. The *exponents* are computed for the *base + power* schemes.

Member State	Population	Status quo	EP-UK 751 (MCC) (u)	EP-UK Optimal (MCC) (n)	EP-UK 678 (MCC) (d)	EP-UK Minimum (L) (n)
Germany	82 064 489	*96*	96	**96**	96	96
France	66 661 621	*74*	83	**82**	80	79
United Kingdom	-	*73*	-	**-**	-	-
Italy	61 302 519	*73*	78	**76**	75	73
Spain	46 438 422	*54*	63	**62**	60	57
Poland	37 967 209	*51*	55	**53**	51	47
Romania	19 759 968	*32*	35	**33**	30	27
Netherlands	17 235 349	*26*	31	**30**	27	25
Belgium	11 289 853	*21*	24	**22**	20	18
Greece	10 793 526	*21*	23	**22**	20	17
Czech Republic	10 445 783	*21*	23	**21**	19	17
Portugal	10 341 330	*21*	23	**21**	19	17
Sweden	9 998 000	*20*	22	**21**	19	17
Hungary	9 830 485	*21*	22	**21**	18	16
Austria	8 711 500	*18*	20	**19**	17	15
Bulgaria	7 153 784	*17*	18	**17**	15	13
Denmark	5 700 917	*13*	16	**15**	13	12
Finland	5 465 408	*13*	16	**14**	13	12
Slovakia	5 407 910	*13*	16	**14**	13	11
Ireland	4 664 156	*11*	14	**13**	12	11
Croatia	4 190 669	*11*	14	**13**	11	10
Lithuania	2 888 558	*11*	11	**11**	9	9
Slovenia	2 064 188	*8*	10	**9**	8	8
Latvia	1 968 957	*8*	10	**9**	8	8
Estonia	1 315 944	*6*	8	**8**	7	7
Cyprus	848 319	*6*	7	**7**	6	6
Luxembourg	576 249	*6*	7	**6**	6	6
Malta	434 403	*6*	6	**6**	6	6
UE-27	445 519 516	*751*	751	**721**	678	640
Total transfer of seats		0	73	43	34	56
Exponent		-	0.796	0.829	0.886	1

Population data based on the Council Decision 2016/2353 of 8 December 2016 amending the Council's Rules of Procedure. See the endnotes 16 & 19 for the formulae used to compute apportionments.

3. THE SYSTEM OF VOTING IN THE COUNCIL OF THE EU

Adopting one of the mathematically motivated, fair and objective systems of allocation of seats in the EP recommended above, will lead to a certain transfer of power in the European Union. In particular, the largest Member States (with the exception of Germany) will increase their number of representatives in the Parliament. Therefore, to preserve the overall **balance of power** in the Union, it is well justified to consider a simultaneous suitable modification of the existing voting system in the **Council**.

The current solution, adopted in December 2007 in Lisbon, is based on the principle of 'double majority': a decision of the Council is taken if it is supported by a coalition, which:

a. is formed by at least 55% of the Member States,
b. represents at least 65% of the population of the Union.

Additionally, a decision is adopted if the supporting coalition consists of all but three (or fewer) countries even if it represents less than 65% of the population of the Union[44].

The case of **Brexit** creates an urgent need to discuss and reconsider these rules[45]. A detailed analysis by Moberg shows that the current system of the 'double majority' is not really double, as the population criterion (b) plays a dominant role[46]. As noted by several authors[47], the existing system is biased in favour of the most and the least populated countries. In particular, the voting power of a typical citizen in these states, measured by the Banzhaf-Penrose index, is larger than the power of a medium-sized state citizen. These disadvantages of any 'double majority' voting system were noted by Lionel Penrose[48] as long ago as in 1952. Working on the problem of voting power, Penrose formulated his square root law and proposed an objective voting system, in which the voting weights are proportional to the square root of the population for each state. A voting system based on the Penrose law was first proposed for the Council of Ministers by Laruelle and Widgrén[49] in 1996 and, independently, by Felsenthal and Machover[50] in 1997.

However, to construct any weighted voting system one has to choose not only the voting weights, but also to fix the quota (threshold) of the qualified majority, which plays a crucial role in the system[51]. In the past, the quotas in the voting systems for the Council had been established subjectively in a bargaining procedure, without an objective justification. A new solution to the problem relates the value of the quota to an optimization procedure: the **optimal quota** is set in such a way that the voting power of every citizen in each Member State is approximately equal[52]. Such a solution is known in the literature as the **Jagiellonian Compromise**[53] and its advantages have been acknowledged by several experts[54]. Its name is related to the fact that it can be considered as an objective and fair compromise between the older Nice voting system, in which the largest Member States suffer a relatively small voting power, and the current 'double majority' voting system, where they seem to have too much power.

The **Jagiellonian Compromise** is a voting system for the **Council of Ministers** consisting of a single criterion only determined by the following two rules:

1. Each Member State is attributed the voting weight proportional to the square root of its population;
2. The decision of the Council is taken if the sum of the weights of members of a coalition supporting it exceeds the quota equal to the arithmetical mean of the sum of the weights and the square root of the total population of the Union.

The quota for the qualified majority is considerably larger than 50% for any size of the voting body of a practical interest. Thus, the voting system is moderately conservative, as it should be. Furthermore, it is transparent: the voting power of each Member State, measured by the Banzhaf-Penrose index, is, up to a high accuracy, proportional to its voting weight. As a crucial advantage of the system one can emphasize its extendibility: if the number of Member States or their populations change, all one needs to do is to set the voting weights according to the rule (1), and adjust the quota according to the rule (2).

Currently, the value of the **optimal quota** equals (approximately) **61.4%**; while after Brexit it would change to **61.6%**. Implementing a new voting system in the Council based on the Jagiellonian Compromise would contribute to an increase in the *a priori* voting power of the medium-sized members of the EU. In a sense, this step would compensate for the losses incurred by these states due to the allocation of seats according to the (Modified) Cambridge Compromise and will contribute to **preservation of the current overall balance of power in the European Union**.

Final recommendation for the voting system in the Council

*We recommend the adoption of the **Jagiellonian Compromise** as the degressively proportional solution for the voting system in the Council of Ministers, counterbalancing the effects of the new apportionment of seats in the European Parliament.*

[1] Consolidated version of the Treaty on European Union, OJ C 326, 26.10.2012.

[2] European Council Decision of 28 June 2013 establishing the composition of the European Parliament, OJ L 181, 29.6.2013, p. 57–58.

[3] Cegiełka, K., 2013, Composition of the European Parliament in the 2009-2014 term. Didactics of Mathematics 9, 25-34.

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[5] Pukelsheim, F., 2014, Proportional Representation – Apportionment Methods and Their Applications. With a Foreword by Andrew Duff MEP. Springer, Cham (CH), p. 166.

[6] However, the concept does not appear in Balinski, ML., Young, HP., 1982, Fair representation in the European Parliament. Journal of Common Market Studies 20, 361-373.

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[11] Bummel, A., 2010, The composition of a parliamentary assembly at the United Nations. Background Paper No. 3. Berlin: Committee for a Democratic UN; see also von Bogdandy, A., 2012, The European Lesson for International Democracy: The Significance of Articles 9–12 EU Treaty for International Organizations. The European Journal of International Law 23, 315-334.

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[14] Grimmett, G., Laslier, J.-F., Pukelsheim, F., Ramírez-González, V., Rose, R., Słomczyński, W., Zachariasen, M., Życzkowski, K., 2011, The allocation between the EU member states of the seats in the European Parliament. Cambridge Compromise. European Parliament Studies, PE 432.760.

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[16] The detailed formulae can be found in Słomczyński & Życzkowski 2012, *op. cit.* Here, we would like to recall only two of them: $A(t) = min\,(m + (t-p)/d\,, M)$ (for *base* + *prop*), and $A(t) = \left(M(t^d - p^d) + m(P^d - t^d)\right)/(P^d - p^d)$ (for *base* + *power*), where *m* and *M* denote the number of seats for the smallest and the largest Member State, with population *p* and *P*, respectively, and $p \le t \le P$, used for computing the numbers in Table 1. and Table 2.

[17] Pukelsheim, F., 2007, A Parliament of degressive representativeness? Preprint No. 015/2007, Institut für Mathematik, Universität Augsburg.

[18] Grimmett *et al.* 2011, *op. cit.*

[19] Here, we define the base + prop method in slightly different manner than in Grimmett *et al*. 2011, *op. cit*. to guarantee fulfilling the boundary conditions: A(p) = m and A(P) = M, as required by the Council. This 'spline' version of the Cambridge Compromise was proposed for the first time in Martínez-Aroza, J., Ramírez-González, V., 2008, Several methods for degressively proportional allocations. A case study. Mathematical and Computer Modelling, 48, 1439–1445. There are some minor differences between this version and the original one.

[20] Słomczyński & Życzkowski 2012, *op. cit.*

[21] Ramírez-González, V., Martínez Aroza, A., Márquez García, M., Spline methods for degressive proportionality in the composition of the European Parliament. Mathematical Social Sciences 63, 114-120.

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[23] Ramírez-González *et al.* 2006, *op. cit.*

[24] Grimmett, G., Oelbermann, K.-F., Pukelsheim, F., 2012, A power-weighted variant of the EU27 Cambridge Compromise. Mathematical Social Sciences 63, 136–140; see also Pukelsheim 2014, *op. cit.,* p. 170., where the method is called the *downgraded-population variant* of the Cambridge Compromise.

[25] Theil, H., Schrage, L., 1977, The apportionment problem and the European Parliament. European Economic Review 9, 247–263.

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[27] Słomczyński & Życzkowski 2012, *op. cit.*

[28] Serafini, P., 2012, Allocation of the EU Parliament seats via integer linear programming and revised quotas. Mathematical Social Sciences 63, 107–113.

[29] Słomczyński & Życzkowski 2010, *op. cit.*

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[32] Kellermann, T., 2012, The minimum-based procedure: A principled way to allocate seats in the European Parliament. Mathematical Social Sciences 63, 102-106.

[33] Using the measure of degressive proportionality introduced in: Dniestrzański, P., 2014, Proposal for measure of degressive proportionality, Procedia - Social and Behavioral Sciences 110, 140–147, one can check that it is possible to order considered schemes from the least to the most degressively proportional: *base + prop < homographic ≈ parabolic < base + power < piecewise linear ≈ linear + hyperbolic*. A similar hierarchy can be reproduced by computing the relative entropy of the population distribution with respect to the seats distribution, measuring in this way the deviation form proportionality, see Lauwers, L., Van Puyenbroeck, T., 2008, Minimally Disproportional Representation: Generalized Entropy and Stolarsky Mean-Divisor Methods of Apportionment, HUB research paper, Brussel.

[34] Pukelsheim 2014, *op. cit.,* p. 168-174.

[35] Grimmett *et al.* 2011, *op. cit.*

[36] Moberg, A., 2012, EP seats: the politics behind the math. Mathematical Social Sciences 63, 78–84.

[37] Grimmett *et al.* 2012, *op. cit.*

[38] Patel, O., Reh, C., 2016, Brexit: The Consequences for the EU's Political System. UCL Constitution Unit Briefing Paper 2.

[39] European Council Decision of 28 June 2013 establishing the composition of the European Parliament, *op. cit.*

[40] Note that this fact seems to be crucial for practical implementation of a given apportionment scheme. For instance, in opinion of the authors of *Report on the composition of the European Parliament with a view to the 2014 elections.* A7-0041/2013, the rapporteurs, Gualtieri and Trzaskowski, both base + prop and parabolic schemes deviated too strongly from the *status quo*. As they wrote: *The implementation [of the 'Cambridge compromise' as the most 'proportional' mechanism respecting degressive proportionality] would trigger a traumatic reallocation of seats, with heavy losses for medium-sized and small Member States and huge increases for larger ones. Furthermore, failure to abolish the 96 upper limit would discriminate against Germany among the large Member States, introducing a steep rise in the population/seats ratio between France and Germany. Among the various possible mathematical formulae for implementing the principle of degressive proportionality the 'parabolic' method is one of the most degressive. It could, in the longer term, be used as a benchmark in the absence of a treaty change, but the redistribution which this model entails would be too drastic to be politically sustainable in a single step. Cf.* note 33.

[41] E.g., the population of Sweden is currently larger than that of Hungary, yet Hungary has one more seat in the European Parliament.

[42] Balinski & Young 1982, *op. cit.*

[43] See Słomczyński & Życzkowski 2012, *op. cit.* for a precise definition of this notion.

[44] Consolidated version of the Treaty on European Union, OJ C 326, 26.10.2012, Article 16(4).

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[47] See [CŻ10]: Kirsch, W., The distribution of power in the Council of Ministers of the European Union, p. 93-107; Pukelsheim, F., Putting citizens first: Representation and power in the European Union, p. 235-253; Słomczyński & Życzkowski 2010, *op. cit.*

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[54] Pukelsheim 2014, *op. cit.,* p. 174-176.

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