EUROPEAN PARLIAMENT

DIRECTORATE-GENERAL FOR INTERNAL POLICIES

POLICY DEPARTMENT C
CITIZENS' RIGHTS AND CONSTITUTIONAL AFFAIRS

Constitutional Affairs

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The allocation between the EU Member States of the seats in the European Parliament

NOTE

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EUROPEAN PARLIAMENT

DIRECTORATE GENERAL FOR INTERNAL POLICIES

POLICY DEPARTMENT C: CITIZENS' RIGHTS AND CONSTITUTIONAL AFFAIRS

CONSTITUTIONAL AFFAIRS

THE ALLOCATION BETWEEN THE EU MEMBER STATES OF THE SEATS IN THE EUROPEAN PARLIAMENT

Cambridge Compromise

NOTE

Abstract

This Note contains the recommendation for a mathematical basis for the apportionment of the seats in the European Parliament between the Member States of the European Union. This is the unanimous recommendation of the Participants in the Cambridge Apportionment Meeting, held at the instigation of the Committee on Constitutional Affairs at the Centre for Mathematical Sciences, University of Cambridge, on 28–29 January 2011.

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EXECUTIVE SUMMARY

The Committee on Constitutional Affairs has commissioned a Symposium of Mathematicians to recommend a mathematical "formula" for the apportionment of the seats of the European Parliament between the Member States. The purpose of the reform is "to eliminate the political bartering which has characterised the distribution of seats so far", and the required formula should be "durable, transparent and impartial to politics".

The apportionment formula is required to allocate a minimum of 6 seats to each Member State, and a maximum of 96 seats. The Parliament-size is to be no greater than 751. The recommended formula should satisfy a condition of degressive proportionality.

The Cambridge Apportionment Meeting took place in January 2011. Its **principal recommendation** is that seats in the European Parliament be apportioned in such a way that each Member State receives a base of 5 seats, and the remaining seats are divided between the Member States in proportion to their populations (subject to the constraints). The recommended "base+prop" formula rounds fractional allocations of seats upwards, thereby guaranteeing a minimum of 6 seats to every Member State. Tables displaying the application of this formula to the current European Union, as well as after accessions by Croatia and Iceland, may be found in Section 9.

Associated with the principal recommendation is a **recommendation** to amend the definition of "degressive proportionality" in order to accommodate mathematical methods for the apportionment of Parliament. The reason is that mathematical methods generally result in allocations failing to satisfy the definition presented by the former Rapporteurs Alain Lamassoure and Adrian Severin in their 2007 Report.

It is further **recommended** that consideration be given to the value of the minimum number of seats, currently 6, in the light of future accessions to the European Union, and that the functioning of the maximum number, currently 96, be reviewed prior to each quinquennial apportionment of Parliament.

The final **recommendation** of the Cambridge Apportionment Meeting is that Eurostat be asked to review the methods used by Member States in calculating their populations.

INTRODUCTION

The author was invited by the Committee on Constitutional Affairs (AFCO) to write a Briefing Note and to organize a Symposium of Mathematicians in order to recommend a mathematical "formula" for the apportionment of the seats of the European Parliament. The background to this Symposium is explained in the following extracts from a note[1] prepared for the Committee on Constitutional Affairs by the Rapporteur, Mr Andrew Duff:

> *The aim of the symposium is to discuss and, if possible, to propose to the Committee on Constitutional Affairs a mathematical formula for the redistribution of the 751 seats in the European Parliament. The formula should be as transparent as possible and capable of being sustained from one Parliamentary mandate to the next.*
>
> *The purpose of the reform is to eliminate the political bartering which has characterised the distribution of seats so far by enabling a smooth reallocation of seats once every five years which takes account of migration, demographic shifts and the accession of new Member States.*

The Cambridge Apportionment Meeting convened on 28–29 January 2011 at the Centre for Mathematical Sciences, University of Cambridge. This is its Report.

[1] Duff, A. *Mathematical Symposium, Cambridge, 28–29 January 2011.* European Parliament, 2011.

1. CAMBRIDGE APPORTIONMENT MEETING

THIS REPORT CONTAINS THE UNANIMOUS RECOMMENDATIONS OF THE PROFESSORS OF MATHEMATICS AND PUBLIC POLICY WHO PARTICIPATED IN THE CAMBRIDGE APPORTIONMENT MEETING.

1.1. Participants

Mathematics

- Prof. Geoffrey Grimmett (University of Cambridge), *Director*
- Prof. Friedrich Pukelsheim (University of Augsburg), *co-Director*
- Prof. Jean-François Laslier (École Polytechnique, Paris)
- Prof. Victoriano Ramírez González (University of Granada)
- Prof. Wojciech Słomczyński (Jagiellonian University, Kraków)
- Prof. Martin Zachariasen (University of Copenhagen)
- Prof. Karol Życzkowski (Jagiellonian University, Kraków)

Public Policy

- Prof. Richard Rose (University of Aberdeen; European University Institute, Florence)

AFCO Committee in attendance

- Mr Andrew Duff MEP (Rapporteur)
- Mr Rafał Trzaskowski MEP (Vice-President)
- Mr Guy Deregnaucourt (Administrator)
- Mr Wolfgang Leonhardt (Administrator)
- Mr Kevin Wilkins (Assistant to Mr Duff)

Students in attendance

- Mr Thomas Kellermann (College of Europe, Natolin, Warsaw)
- Ms Kai-Friederike Oelbermann (University of Augsburg)

1.2. Terms of reference

These are the Terms of Reference for the Briefing Note, received from the Committee on Constitutional Affairs.

Background

The European Parliament is discussing a proposal for a modification of the Act concerning the election of the Members of the European Parliament by direct universal suffrage of 20 September 1976.

On 19 May 2010 the Rapporteur on the Constitutional Affairs Committee of the European Parliament, Mr Andrew DUFF, presented his working document on the proposal and on 5 July 2010 the draft report on the same issue.

One of the core issues of the report concerns the principle of degressive proportionality for the distribution of seats.

The problem of fitting everyone in fairly beneath the ceiling of 751 according to the

principle of degressive proportionality lies directly at the door of the European Parliament which, under the Treaty of Lisbon, acquires the right (and the duty) to initiate changes in the composition of the House during each mandate. Parliament will also be responsible for proposing the temporary addition of MEPs from accession states during a five-year mandate.

Aim and content of the briefing note
Against this background, the requested briefing note shall identify a mathematical formula for the distribution of seats which will be *durable*, *transparent* and *impartial to politics*.[2]

Methodology and language
The analysis to be undertaken shall be objective and all sources of information must be indicated.

The note shall be drawn up in English.

Delivery conditions
The final version of the briefing note must be handed over to the European Parliament at the latest on 31 March 2011.

[2] Italics by the current author.

2. CONSTRAINTS ON THE ALLOCATION METHOD

This Report takes note of the following constraints:

- the Parliament size shall not exceed 751,
- there shall be a minimum threshold of 6 seats per Member State,
- no Member State shall receive more than 96 seats,
- no smaller State shall receive more seats than a larger State,
- the allocation shall respect the principle of "degressive proportionality".

We were advised by the AFCO representatives in attendance that the first three constraints are not absolutely rigid, but nevertheless there exists a general expectation in Parliament that its total size should not be less than 751, and that the smallest States should receive an allocation not greater than 6 seats.

The issue of "degressive proportionality" is addressed in detail in Section 3 of this Report.

In reaching our conclusions, we have taken into account the following additional observations concerning the general structure of the European Parliament:

- the EU has currently 27 Member States,
- the smallest population (as published officially by Eurostat) is currently 412 970, and the largest 81 802 257,
- future accessions may include a number of States with a spread of sizes,
- there will be migration and demographic changes,
- Member States' population figures (as published by Eurostat) will be used as input to the formula.

3. DEGRESSIVE PROPORTIONALITY

3.1. Background

We discussed the issue of degressive proportionality in depth, with specific reference to the European Parliament Resolution on "Proposal to amend the Treaty provisions concerning the composition of the European Parliament" adopted on 11 October 2007 (Report of the Rapporteurs Alain Lamassoure and Adrian Severin to the Committee on Constitutional Affairs)[3]. In Paragraphs 4 and 6 of the Motion, it is written:

> 4. *[The European Parliament] Notes that Article [9a] of the Treaty on European Union as incorporated in the draft amending treaty provides for a framework comprising an overall ceiling of 750, a maximum of 96 for the most populous Member State and a minimum of 6 for the least populous Member State, and that it lays down the principle of representation of European citizens in accordance with degressive proportionality, without defining this term in any more precise way;*
> 6. *[The European Parliament] Considers that the principle of degressive proportionality means that the ratio between the population and the number of seats of each Member State must vary in relation to their respective populations in such a way that each Member from a more populous Member State represents more citizens than each Member from a less populous Member State and conversely, but also that no less populous Member State has more seats than a more populous Member State.*

In Paragraph 10 of the "Explanatory Statement" of the above Resolution, it is written:

> 10. *Within this context, how to go forward? The ideal alternative would be to agree on an undisputed mathematical formula of "degressive proportionality" that would ensure a solution not only for the present revision but for future enlargements or modifications due to demographic changes.*

Thus, "degressive proportionality" is to include two requirements:

DP1. no smaller State shall receive more seats than a larger State,

DP2. the ratio population/seats shall increase as population increases.

Condition DP1 is easy to accept on the basis of rationality, but Condition DP2 poses a serious practical difficulty, and has in addition been violated in recent Parliamentary apportionments. It has been noted[4] that there are hypothetical instances of apportionment for which *there exists no solution satisfying both DP1 and DP2.* It is in the nature of mathematicians to distil the essence of an assertion, and thus the following example is included here. Imagine a Parliament comprising 105 seats that is to contain representatives of just five States as follows.

[3]European Parliament Resolution on "Proposal to amend the Treaty provisions concerning the composition of the European Parliament" adopted on 11 October 2007 (INI/2007/2169).

[4] Noted in reference [7] at the end of the Report. See also references [9,12,13].

Member State	*Population*	*Apportionment*
Greece	11 305 118	21
Belgium	10 839 906	21
Portugal	10 637 713	21
Czech Republic	10 506 813	21
Hungary	10 014 324	21
Totals	*53 303 874*	*105*

Each State must receive exactly 21 seats if the two conditions are to be satisfied. Imagine now that the given Parliament-size is 106 rather than 105. If Condition DP1 is to be satisfied, the extra seat must be allocated to Greece. However, this violates Condition DP2, since

$$513\,869 = \frac{11\,305\,118}{22} < \frac{10\,839\,906}{21} = 516\,186.$$

That is, a MEP from the largest Member State, Greece, represents fewer citizens than a MEP from a smaller Member State, Belgium.

This difficulty arises in situations where there are a number of Member States having similar populations. While the above example is hypothetical, it illustrates the reason why, in typical apportionments by mathematical means, there are violations of Condition DP2.

Conditions DP1 and DP2, acting together, can force States with similar populations to have equal numbers of seats. As further States become similar in size through, for example, accession, migration, or demographic change, they may be coerced into near equality. One may note that this occurs because of pair-wise comparisons between pairs of States when ordered by population-size.

3.2. Our recommendation

Two approaches to degressive proportionality were discussed.

A. Adopt a method whose outcomes satisfy Condition DP2 but with a possibly reduced Parliament-size.
B. Propose a change to the Lamassoure–Severin definition of degressive proportionality that lies within existing law and allows greater flexibility and transparency.

A method ("iterative interpolation") achieving A was summarized at the Symposium, and is included in Section 6.2. We noted that this method, while mathematically sound, does not generally achieve the given Parliament-size, and secondly that its formulation is markedly less transparent than the method ultimately recommended and described in Section 4.

It was agreed that Alternative B is the better way forward, and can be achieved easily via a minor amendment of the wording of the European Parliament's Resolution on "Proposal to amend the Treaty provisions concerning the composition of the European Parliament"[5]. We recommend that degressive proportionality be interpreted by amending Paragraph 6 of the Resolution through the addition of the bold text as follows:

[5] European Parliament Resolution on "Proposal to amend the Treaty provisions concerning the composition of the European Parliament" adopted on 11 October 2007 (INI/2007/2169).

6. *[The European Parliament] Considers that the principle of degressive proportionality means that the ratio between the population and the number of seats of each Member State* ***before rounding to whole numbers*** *must vary in relation to their respective populations in such a way that each Member from a more populous Member State represents more citizens than each Member from a less populous Member State and conversely, but also that no less populous Member State has more seats than a more populous Member State.*

RECOMMENDATION 1 IS THAT THE EUROPEAN PARLIAMENT ADOPT THIS REVISED DEFINITION OF DEGRESSIVE PROPORTIONALITY.

Recommendation 1 would not have been timely at the adoption in 2007 of the European Parliament Resolution on "Proposal to amend the Treaty provisions concerning the composition of the European Parliament", since seats were then allocated by negotiation rather than by formula. Recommendation 1 may be viewed as a minor adaptation of the 2007 resolution, motivated by the search for robustness.

4. CAMBRIDGE COMPROMISE

This Section contains our recommendation for the apportionment of the seats in the European Parliament.[6]

4.1. Base+prop method

The so-called base+prop method proceeds in two stages. At the first stage, a fixed *base* number of seats is allocated to each Member State. At the second stage, the remaining seats are allocated to States in proportion to their population-sizes (subject to rounding, and capping at the maximum). In order to achieve the given Parliament-size, one introduces a further ingredient called the *divisor.*

The *base+prop* method is formulated as follows[7]:

1. assign to each Member State a fixed number of seats, called the *base* and denoted as *b,*
2. for a given *divisor d,* assign to a Member State with population *p* a further *quotient p/d,* resulting in the seat share $b + p/d$,
3. perform a rounding of the seat share $b + p/d$ into a whole seat number $[b + p/d]$,
4. if the seat number $[b + p/d]$ exceeds the maximum allocation, replace it by this maximum,
5. adjust the divisor *d* in such a way that the sum of the seat numbers of all Member States equals the given Parliament-size.[8]

RECOMMENDATION 2 IS TO ADOPT A BASE+PROP METHOD OF APPORTIONMENT.

4.2. Choice of base and rounding method

RECOMMENDATION 3 IS TO SET THE BASE *b* = 5 AND TO USE *ROUNDING UPWARDS.*

4.3. Cambridge Compromise apportionments in practice

In Section 9 are presented Tables indicating apportionments of the European Parliament following the Cambridge Compromise, for the current 27 Member States, and with the additions of Croatia and Iceland in that order. Population figures are taken from the Eurostat website[9].

[6] Our recommendation is named in harmony with the Jagiellonian Compromise proposal for voting within the Council of the European Union. See references [8,10] at the end of the Report.
[7] For a discussion of this method see reference [9], where it is referred to as the "fix+prop" method.
[8] There will generally be a range of possible values for the final divisor; see Question 8.7 in Section 8.
[9] http://epp.eurostat.ec.europa.eu/

5. APPORTIONMENT METHODS

5.1. Methods of apportionment

We concentrated on apportionment methods comprising the following two choices: of an *apportionment function A,* and of a *rounding method.*

1. Choose an apportionment function *A* sufficiently general to be capable of accommodating the given constraints on minimum, maximum, and Parliament-size,
2. allocate to a Member State with population *p* the fractional number *A(p)* seats,
3. round *A(p)* to obtain a whole number [*A(p)*],
4. adjust the function in such a way as to satisfy the given constraints.

General accounts of such functions may be found in articles contained in references [3,4] at the end of the Report.

The apportionment function is to be chosen in such a way that allocations are non-decreasing in population, and satisfy degressive proportionality (and possibly other criteria allied to principled approaches to apportionment). The specific functions considered at the Cambridge Apportionment Meeting satisfy the amended version of degressive proportionality proposed in Section 3.2, but each can result in violations of the Lamassoure–Severin (2007) condition. For this reason, we resolved to recommend the amended version of degressive proportionality.

We considered two categories of apportionment function, basically those comprising linear elements, and those with non-linear elements.

- Non-linear functions, including parabolic and power functions[10].
  Advantages: smoothness (in the mathematical sense).
  Disadvantages: non-linear (thus, with a diminished potential for proportionality), unprincipled use of a power, non-robustness with respect to certain population distributions, non-transparent, relative difficulty of computation.
- Linear functions, including base+prop and spline methods[11].
  Advantages: transparent, greater potential for proportionality, ease of computation.
  Disadvantages: non-smoothness of allocation function at the maximum.

We noted that, as the European Union is progressively enlarged (the constraints being unchanged), the non-linear allocation functions become closer to linear.

Having taken this analysis into consideration, we resolved to restrict attention to allocation functions of linear type. The simplest of these applies a single linear function, subject to adjustments that guarantee the minimum and maximum constraints.

Following a discussion of the merits of base+prop and other linear-type methods such as spline methods, we resolved that base+prop is amongst the most transparent methods, and is preferable to others on the grounds of degressive proportionality.

We noted that the base+prop method can be interpreted as one in which the *base* is an allocation to Member States, and the remaining seats *(prop)* are proportional to population (subject to capping at the maximum). We felt that this interpretation might find favour with the Committee on Constitutional Affairs, in view of the founding principles of the EU, enshrined in the Treaty, that the Union is made up both of Member States (enjoying

[10] See references [11,12,13] at the end of the Report.
[11] See references [5,7,9].

equality in international law) and of citizens (enjoying democratic equality). Notwithstanding, we recommend the base+prop method on the basis of the remit of Section 1.2, in particular for its durability, transparency, impartiality, and its adherence to degressive proportionality.

5.2. Rounding methods

In many situations in life, one may be confronted with a fraction *x* when a whole number is required. There are a variety of methods for 'rounding' *x* to a nearby whole number. This situation is commonplace in apportionment problems given the integral nature of the unit being distributed.

We concentrate here on the three principal methods of rounding:

- *rounding downwards*: for example, 5.1 ↓ 5, 5.9 ↓ 5,
- "*standard rounding*" *to the nearest whole number*: 5.1 ↓ 5, 5.9 ↑ 6,
- *rounding upwards*: 5.1 ↑ 6, 5.9 ↑ 6.

There is an extensive literature on rounding methods for apportionment, especially in the context of the US House of Representatives[12]. Previous analysis indicates that, other things being equal, downwards (respectively, upwards) rounding tends to favour larger (respectively, smaller) States, and standard rounding is generally regarded as being fairly neutral to State size.

5.3. Choosing the base and rounding method

Implementation of the base+prop method requires a choice of base. This choice is informed by the demand that the smallest States have a minimum of 6 seats, and by the principle of degressive proportionality. Since the base allocation is made without regard to population-size,

- a larger base tends to favour smaller States,
- a smaller base tends to favour larger States.

The relationship between the smaller and larger States is relevant to choice of both base and rounding method, and these two choices are therefore inter-related. The relevant factors are (a) the minimum, and (b) degressive proportionality. In the interests of transparency, we favoured a whole number base rather than a fraction. Two candidate methods propose themselves given the minimum number 6 of seats.

- *6 + standard.* Base = 6, with standard rounding to the nearest whole number.
- *5 + upwards*. Base = 5, with rounding upwards to a whole number.

Each of these methods guarantees a minimum of 6 seats to all States, and is degressively proportional in the amended sense proposed in Section 3.2.

We argued as follows in reaching Recommendation 3 (namely, to adopt the method *5 + upwards*). Since each of the above methods satisfies the amended form of degressive proportionality, we concentrated on the durability of the minimum. We were advised by the AFCO representatives in attendance of an expectation that the smallest States in the current EU would indeed be allocated exactly 6 seats. With the current population figures, the smallest four Member States are as follows:

[12] See reference [5] for a survey and critique, and also reference [14].

	Member State	*Population*	*5 + upwards*	*6 + standard*	*Now*
24	Estonia	1 340 127	7	8	6
25	Cyprus	803 147	6	7	6
26	Luxembourg	502 066	6	7	6
27	Malta	412 970	6	6	6

Against each State is reported its current allocation ('Now'), and its hypothetical allocations under the two methods under consideration.

Suppose that the population of Malta increases by 8000, and the other populations remain unchanged. The amended Table is as follows:

	Member State	*Population*	*5 + upwards*	*6 + standard*	*Now*
24	Estonia	1 340 127	7	8	6
25	Cyprus	803 147	6	7	6
26	Luxembourg	502 066	6	7	6
27	Malta	420 970	6	7	6

By reason of the increase in its population, Malta gains a seventh seat under the method *6 + standard.* We viewed this as a sign of non-robustness in the minimum, and for this reason we have recommended the use of *5 + upwards.*[13]

[13] After the Symposium was completed, one of our tentative conclusions was confirmed, namely that *5 + upwards* results inevitably in the same allocation as *6 + downwards,* and indeed as *5.5 + standard.* See reference [6].

6. FURTHER DISCUSSION

6.1. Base+prop for the mathematician

A mathematical formulation of the base+prop method of apportionment is presented next. Let *m* be the minimum and *M* the maximum number of seats per Member State, and *H* the target Parliament-size. Write *b* for the *base,* and *d* for the free parameter called the *divisor.* The allocation function *A* is given effectively by

$$A(p) = \min\left\{b + \frac{p}{d}, M\right\},$$

where *p* denotes the population of a State. The total number of seats allocated to *n* Member States with populations $p_1,\ p_2, \ldots, p_n$ is

$$T(d) = [A(p_1)] + [A(p_2)] + \ \ldots \ + [A(p_n)],$$

where $[x]$ is a rounding of the fraction *x,* and the divisor *d* is chosen[14] in such a way that $T(d) = H$.

The European Parliament has currently *m* = 6, *M* = 96, *H* = 751. We recommend taking the base *b* = 5 and *rounding upwards.*

Does there invariably exist a value of the divisor *d* that results in the *exact* value of the given Parliament-size? This general question has received attention in the mathematical literature of apportionment (see reference [5] at the end of the Report, for example). An issue can arise as a result of unlikely but conceivable coincidences in the factors of population-sizes, but these are very rare in an apportionment problem of the scale of the European Parliament. We feel that this possibility can be disregarded[15].

6.2. The iterative-interpolation method of apportionment

Details are presented of a further apportionment method of linear type. This is included in this Report as an example of a method that results invariably in an allocation satisfying the Lamassoure–Severin degressive proportionality conditions DP1 and DP2 (see Section 3.1). We do not recommend the adoption of this method, for reasons including its relative lack of transparency. We do not believe that adherence to the Lamassoure–Severin (2007) definition of degressive proportionality should emerge as a greater priority than transparency.

First, we review the spline method[16]. It has allocation function

$$A(p) = \min\left\{m + \frac{p - p_{\min}}{d}, M\right\}$$

[14] Base+prop is an example of a so-called "divisor method". For the sake of clarity for the lay reader, we might take the initial divisor as the total population of the European Union divided by the maximum number of seats minus the base allocation *nb.* This choice does not take into account the capping of allocations at the maximum, and is for illustration only.
[15] See reference [5] for an analysis of "ties".
[16] Readers are referred to reference [7] for a fuller discussion.

where $p_{\min}$ is the population of the smallest Member State. It differs from the base+prop method in that it is designed to allocate the given minimum to the smallest State *before rounding.*

The iterative-interpolation method is summarized as follows.

1. The spline method is applied to the entire set of Member States, and the largest value *k* is found such that the smallest *k* States have allocations satisfying the Lamassoure–Severin (2007) conditions. The allocations to the first *k* States are *fixed.*
2. The allocation to the (*k*+1)th State is reduced by one and then *fixed.*
3. The spline method is now applied to the (*k*+1)th and larger States (with adjusted values of the minimum and of $p_{\min}$), and the above steps are iterated until all States have received their fixed allocations.

It may be checked that apportionment by the method of iterated interpolation:

- inevitably satisfies the Lamassoure–Severin (2007) definition of degressive proportionality,
- does not necessarily achieve the exact Parliament-size, but may have smaller total.

6.3. Choosing the minimum and the base

The minimum number of seats per Member State is currently set to 6; how should this minimum vary as the number of Member States changes? We were not asked to report on this question, but nevertheless we embarked on a discussion as an aid to understanding the role of the minimum.

We considered ways in which the minimum might be required to vary. As the minimum varies, so must the base. Two possible approaches are:

A. determine a minimum that varies with the number of States, and then choose an appropriate (possibly fractional) base,
B. choose a base that decreases formulaically as the number of States increases, and observe how the minimum varies, in the presence of a given rounding method.

We felt that, regardless of which approach is adopted, the issue of transparency dictates that the minimum be clearly defined.

Scheme A

Our approach to Scheme A is as follows. In the current setting, there are 27 Member States with a minimum of 6 seats, so that the total number of seats involved in the minimum is $6 \times 27 = 162$, a proportion of $162/751 \approx 22\%$. Our favoured approach, having heard the advice of the Rapporteur, is to cap this proportion at a suitable figure, and to choose the minimum as large as possible subject to this cap. The effect of a cap of 25% would be as follows: a minimum of 6 seats with 27–31 Member States, a minimum of 5 with 32–37 Member States, and so on. The adoption of a proportion lower than 25% would result in a swifter reduction of the minimum.

At the transition points, a number of seats are transferred out of the base allocation into the proportional allocation. This has the effect of adjusting the relationship between the smaller and the larger Member States, since more seats are distributed according to population.

As the minimum varies, so must the base. We considered two ways in which the base could follow the minimum.

A1. The base is set to one less than the minimum. Combined with upwards rounding, this ensures that the minimum constraint is achieved in any situation (as currently pertains amply) in which some State has population-size smaller than the divisor.
A2. The base is set to the smallest value (which will generally be *fractional)* such that the smallest Member State receives the minimum number of seats exactly. (For illustration, subject to the current population figures and rounding upwards, this results in a base of approximately 4.5.)

We discussed without clear conclusion the relative merits of the simplicity, and hence transparency, of Method A1, and the mathematical appeal of an implicit definition such as that of Method A2. If transparency is paramount then Method A1 should be adopted.

Scheme B

In Scheme B, the base b is determined as a function of the number n of Member States. We discussed formulae of the type $b = 135/n$, which is in agreement with the Cambridge Compromise recommendation to take $b = 5$ when $n = 27$. Taken together with upwards rounding and with current population figures, this would imply a minimum of 6 seats in the current European Union and for the two enlargements shown in Section 9.

This approach has the attractive property that the *base,* being generally fractional, decreases in a more gradual fashion. This contrasts with the downward unit-steps of Scheme A1. On the other hand, the *minimum* allocation descends in a manner that may be considered over-sensitive to the smallest population-size. This approach may, therefore, lie beyond the spirit of the current terms of reference.

RECOMMENDATION A IS THAT DUE CONSIDERATION BE GIVEN TO THE MANNER IN WHICH THE MINIMUM, CURRENTLY 6, AND BASE SHOULD VARY IN THE LIGHT OF FUTURE CHANGES IN THE NUMBER OF MEMBER STATES OF THE EUROPEAN UNION.

6.4. Choosing the maximum

The base+prop method takes account of both Member States and Citizenry: each Member State receives 5 seats of right, plus one or more further seats in proportion to its population. This proportionality is valid over the entire range of populations except where the maximum of 96 is invoked. The maximum of 96 is dictated within our terms of reference. For any Member State sufficiently large that its allocation is capped at 96, there is a loss of proportionality. The effect of the maximum constraint is highlighted by the analysis of this Report. As the architecture of the European Union develops through accessions, the significance of the maximum will diminish until it ceases to be operative.

RECOMMENDATION B IS THAT THE EUROPEAN PARLIAMENT SHOULD REVIEW THE MANNER OF FUNCTIONING OF THE MAXIMUM CONSTRAINT ON NUMBER OF SEATS, CURRENTLY 96, PRIOR TO FUTURE APPORTIONMENTS.

6.5. Official population statistics

Census data on population is usually collected only once a decade; hence, official data about the current population is an estimate based on updates of this data. Both the year of the census and the methods of updating can vary between countries, thus introducing inconsistencies between countries. There can also be different national methods of

determining who is counted as a resident. Census data is key to the determination of the apportionment of the European Parliament.

We were told by the AFCO representatives that the Commission is considering proposing a statistics regulation which would allow Eurostat to verify more accurately than at present the consistency and comparability of national data.

RECOMMENDATION C IS THAT THE EUROPEAN COMMISSION BE ENCOURAGED TO ENSURE THAT EUROSTAT REVIEWS THE METHODS USED BY MEMBER STATES IN CALCULATING THEIR CURRENT POPULATIONS, IN ORDER TO ENSURE ACCURACY AND CONSISTENCY.

7. SUMMARY OF RECOMMENDATIONS

To the Committee on Constitutional Affairs, and hence where appropriate to the European Parliament, Council, and Commission:

Principal recommendations

1. **Adopt the revised definition of degressive proportionality proposed in Section 3.2.**
2. **For future apportionments of the European Parliament, a base+prop method should be employed.**
3. **The base should be 5, and fractions should be rounded upwards.**

Further recommendations

A. **Due consideration should be given to the manner in which the minimum, currently 6, and base should vary in the light of future changes in the number of Member States in the European Union.**

B. **The European Parliament should review the manner of functioning of the maximum constraint on number of seats, currently 96, prior to future apportionments.**

C. **The Commission should be encouraged to ensure that Eurostat reviews the methods used by Member States in calculating their current populations, in order to ensure accuracy and consistency.**

8. QUESTIONS ARISING

1. **Why have you recommended base 5 with rounding up, rather than an alternative such as base 6 and standard rounding to the nearest whole number?**
   Answer: See Section 5.3. The recommended method is more robust with respect to the minimum.

2. **Why have you not recommended a method with a greater degree of proportionality, with States allocated fewer than 6 seats being raised to 6?**
   Answer: This refers perhaps to the divisor method that allocates seats proportionally to population (subject to rounding and capping) while raising the allocation where necessary to 6. States with population-sizes that are insufficient to merit 6 seats are raised to exactly 6. The effect is that a substantial number of Member States receive exactly the minimum number 6 of seats. Strict proportionality is reinforced for the larger States where uncapped, and the prominence of the maximum constraint tends to increase. We viewed the decrease in allocations to smaller States as being in tension with the spirit of degressive proportionality.

3. **Why do you cap the allocation to the largest Member State?**
   Answer: Because a maximum of 96 was included in our terms of reference.

4. **Why have you not allocated the non-base seats proportionally to population-size, over the entire range of Member States including the largest?**
   Answer: Given the upper cap of 96, this approach would result in a substantially reduced Parliament-size, and incidentally, a large number of Member States would receive smaller allocations than under the Cambridge Compromise. There is some freedom in the choice of such a method. If one plots a straight line of seats against population, with the smallest and largest States receiving 6 and 96 seats respectively, the ensuing Parliament-size is 703 using standard rounding. Furthermore, 18 Member States receive smaller allocations than under the Cambridge Compromise.

5. **A non-linear allocation function might be a smoother approach to the capping at the maximum. Why have you not recommended such a function?**
   Answer: We considered the use of linear functions to be more transparent than the use of non-linear functions, and to be closer to the concept of proportionality. We noted that the capping at 96 disturbs the linearity at the upper end, and we regretted this from a mathematical point of view since we would have preferred to use a smooth allocation function. However, the maximum of 96 was dictated by our terms of reference, and we considered it preferable to retain linearity for Member States not at the maximum, rather than disturb it throughout the entire list.

6. **How robust is the proposed method with respect to the possible accession of a large State?**
   Answer: The proposed method is very robust in this regard. An acceding large Member State would attract a substantial allocation of seats. Fewer seats would be available to the other Member States, and this would tend to render inoperative the

capping of the allocation to the largest Member States. This conclusion is supported by preliminary calculations based on current population figures.

7. What happens when a new State accedes to the Union during a legislative period?

Answer: The Parliament could be enlarged temporarily to accommodate the number of further seats allocated to the new State. If the divisor was d, and the new population p, this would amount to [*b + p/d*] *seats (capped, if appropriate, at the maximum). For this purpose, a divisor should be determined and published at the time of every apportionment, and the number of seats for an acceding State settled within the Treaty of Accession.*[17]

[17] As noted in Footnote 8, there is generally a range of values for the divisor that result in the same seat allocations. The divisor should be published at the time of an apportionment.

9. APPORTIONMENT TABLES: 27, 28, AND 29 STATES

9.1. Cambridge Compromise with 27 Member States

	Member State	*Population*[18]	*Base*	+ *Quot.* $\frac{\text{Population}}{819\,000}$ →	***Seats***	*Popn/Seats Before Rounding*	*Popn/Seats After Rounding*	*Now*
1	Germany	81 802 257	5	+ 99.9 →	**96**	852 106.8[19]	852 106.8	99
2	France	64 714 074	5	+ 79.02 →	**85**	770 259.3	761 342.0	74
3	United Kingdom	62 008 048	5	+ 75.7 →	**81**	768 264.0	765 531.5	73
4	Italy	60 340 328	5	+ 73.7 →	**79**	766 950.8	763 801.6	73
5	Spain	45 989 016	5	+ 56.2 →	**62**	752 036.4	741 758.3	54
6	Poland	38 167 329	5	+ 46.6 →	**52**	739 643.2	733 987.1	51
7	Romania	21 462 186	5	+ 26.2 →	**32**	687 772.5	670 693.3	33
8	Netherlands	16 574 989	5	+ 20.2 →	**26**	656 745.2	637 499.6	26
9	Greece	11 305 118	5	+ 13.8 →	**19**	601 222.1	595 006.2	22
10	Belgium	10 839 905	5	+ 13.2 →	**19**	594 438.5	570 521.3	22
11	Portugal	10 637 713	5	+ 12.99 →	**18**	591 356.6	590 984.1	22
12	Czech Republic	10 506 813	5	+ 12.8 →	**18**	589 315.9	583 711.8	22
13	Hungary	10 014 324	5	+ 12.2 →	**18**	581 298.7	556 351.3	22
14	Sweden	9 340 682	5	+ 11.4 →	**17**	569 380.7	549 451.9	20
15	Austria	8 375 290	5	+ 10.2 →	**16**	550 056.4	523 455.6	19
16	Bulgaria	7 563 710	5	+ 9.2 →	**15**	531 334.8	504 247.3	18
17	Denmark	5 534 738	5	+ 6.8 →	**12**	470 724.2	461 228.2	13
18	Slovakia	5 424 925	5	+ 6.6 →	**12**	466 706.8	452 077.1	13
19	Finland	5 351 427	5	+ 6.5 →	**12**	463 965.8	445 952.2	13
20	Ireland	4 467 854	5	+ 5.5 →	**11**	427 330.9	406 168.5	12
21	Lithuania	3 329 039	5	+ 4.1 →	**10**	367 250.6	332 903.9	12
22	Latvia	2 248 374	5	+ 2.7 →	**8**	290 290.0	281 046.8	9
23	Slovenia	2 046 976	5	+ 2.5 →	**8**	272 953.4	255 872.0	8
24	Estonia	1 340 127	5	+ 1.6 →	**7**	201 939.0	191 446.7	6
25	Cyprus	803 147	5	+ 0.98 →	**6**	134 291.1	133 857.8	6
26	Luxembourg	502 066	5	+ 0.6 →	**6**	89 446.6	83 677.7	6
27	Malta	412 970	5	+ 0.5 →	**6**	75 027.7	68 828.3	6
	Total	*501 103 425*	*135*		***751***			*754*

Each State receives one non-base seat for every 819 000 *citizens or part thereof. Population/seat ratios are strictly decreasing before rounding, but there are four violations after rounding.*

[18] Population figures are taken from the Eurostat website (OJ 22.12.2010 L 338/47).

[19] Germany has been capped at the maximum 96, and this ratio calculated accordingly.

Notes

1. The divisor 819 000 is such that the seat total is 751. (See Section 4.1).
2. The calculation is illustrated by the case of Sweden. With the divisor 819 000, Sweden gains 5 base seats plus 9 340 682/819 000 ≈ 11.4 further seats. The total is 16.4, which is rounded upwards to 17.
3. If Croatia were to accede during the legislative period, Parliament could be temporarily enlarged by 5 + 6 = 11 seats (since 4 425 747/819 000 ≈ 5.4 → 6).

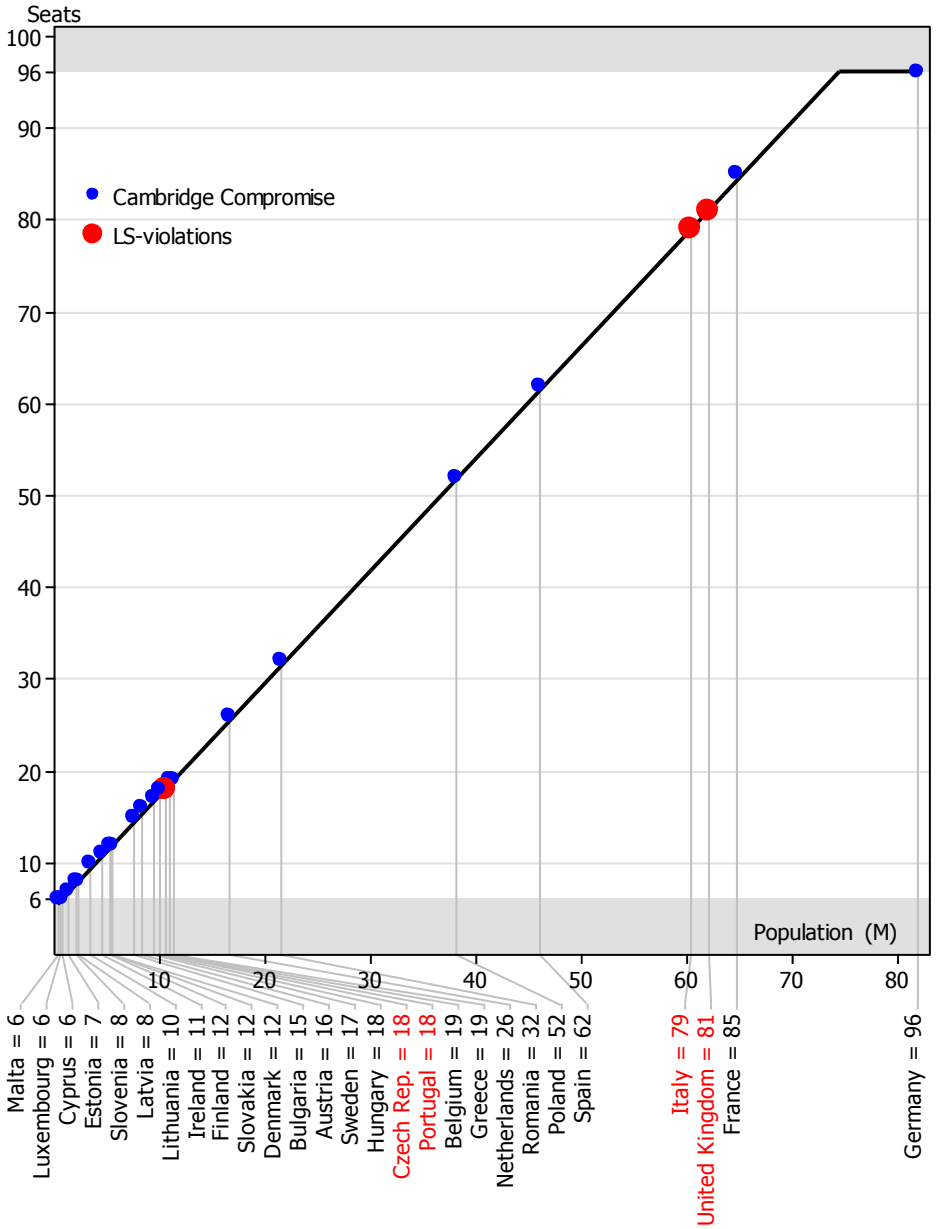

Figure 1. The Cambridge Compromise allocations with 27 Member States. States marked in red have allocations in violation of Condition DP2 of Section 3.1 (considered in order of decreasing population-size). The black line indicates the allocation function before rounding (with capping at the maximum).

9.2. Cambridge Compromise including Croatia

	Member State	*Population*[20]	*Base*	+ *Quot.* →	***Seats***	*Popn/Seats*	*Popn/Seats*	*Now*
				$\frac{\text{Population}}{835\,000}$		*Before Rounding*	*After Rounding*	
1	Germany	81 802 257	5	+ 97.97 →	**96**	852 106.8	852 106.8	99
2	France	64 714 074	5	+ 77.5 →	**83**	784 395.1	779 687.6	74
3	United Kingdom	62 008 048	5	+ 74.3 →	**80**	782 326.0	775 100.6	73
4	Italy	60 340 328	5	+ 72.3 →	**78**	780 964.4	773 593.9	73
5	Spain	45 989 016	5	+ 55.1 →	**61**	765 505.5	753 918.3	54
6	Poland	38 167 329	5	+ 45.7 →	**51**	752 668.1	748 379.0	51
7	Romania	21 462 186	5	+ 25.7 →	**31**	699 020.8	692 328.6	33
8	Netherlands	16 574 989	5	+ 19.9 →	**25**	666 993.9	662 999.6	26
9	Greece	11 305 118	5	+ 13.5 →	**19**	609 799.8	595 006.2	22
10	Belgium	10 839 905	5	+ 12.98 →	**18**	602 822.4	602 216.9	22
11	Portugal	10 637 713	5	+ 12.7 →	**18**	599 653.2	590 984.1	22
12	Czech Republic	10 506 813	5	+ 12.6 →	**18**	597 554.9	583 711.8	22
13	Hungary	10 014 324	5	+ 11.99 →	**17**	589 313.5	589 077.9	22
14	Sweden	9 340 682	5	+ 11.2 →	**17**	577 068.1	549 451.9	20
15	Austria	8 375 290	5	+ 10.03 →	**16**	557 227.5	523 455.6	19
16	Bulgaria	7 563 710	5	+ 9.1 →	**15**	538 023.2	504 247.3	18
17	Denmark	5 534 738	5	+ 6.6 →	**12**	475 966.1	461 228.2	13
18	Slovakia	5 424 925	5	+ 6.5 →	**12**	471 859.1	452 077.1	13
19	Finland	5 351 427	5	+ 6.4 →	**12**	469 057.4	445 952.2	13
20	Ireland	4 467 854	5	+ 5.4 →	**11**	431 646.5	406 168.5	12
21	Croatia	4 425 747	5	+ 5.3 →	**11**	429 671.8	402 340.6	–
22	Lithuania	3 329 039	5	+ 3.99 →	**9**	370 433.5	369 893.2	12
23	Latvia	2 248 374	5	+ 2.7 →	**8**	292 275.1	281 046.8	9
24	Slovenia	2 046 976	5	+ 2.5 →	**8**	274 707.7	255 872.0	8
25	Estonia	1 340 127	5	+ 1.6 →	**7**	202 897.6	191 446.7	6
26	Cyprus	803 147	5	+ 0.96 →	**6**	134 714.3	133 857.8	6
27	Luxembourg	502 066	5	+ 0.6 →	**6**	89 634.2	83 677.7	6
28	Malta	412 970	5	+ 0.5 →	**6**	75 159.6	68 828.3	6
	Total	*505 529 172*	*140*		***751***			*754*

Each State receives one non-base seat for every 835 000 *citizens or part thereof. Population/seat ratios are strictly decreasing before rounding, but there are two violations after rounding.*

1. The divisor 835 000 is such that the seat total is 751.
2. If Iceland were to accede during the legislative period, Parliament could be temporarily enlarged by 5 + 1 = 6 seats (since 317 630/835 000 ≈ 0.4 → 1).

[20] Eurostat 1.1.2011.

9.3. Cambridge Compromise including Croatia and Iceland

	Member State	*Population*[21]	*Base*	+	*Quot.* $\frac{\text{Population}}{844\,000}$	→	***Seats***	*Popn/Seats Before Rounding*	*Popn/Seats After Rounding*	*Now*
1	Germany	81 802 257	5	+	96.9	→	**96**	852 106.8	852 106.8	99
2	France	64 714 074	5	+	76.7	→	**82**	792 332.1	789 196.0	74
3	United Kingdom	62 008 048	5	+	73.5	→	**79**	790 221.0	784 912.0	73
4	Italy	60 340 328	5	+	71.5	→	**77**	788 831.8	783 640.6	73
5	Spain	45 989 016	5	+	54.5	→	**60**	773 062.9	766 483.6	54
6	Poland	38 167 329	5	+	45.2	→	**51**	759 973.0	748 379.0	51
7	Romania	21 462 186	5	+	25.4	→	**31**	705 317.1	692 328.6	33
8	Netherlands	16 574 989	5	+	19.6	→	**25**	672 724.1	662 999.6	26
9	Greece	11 305 118	5	+	13.4	→	**19**	614 586.0	595 006.2	22
10	Belgium	10 839 905	5	+	12.8	→	**18**	607 499.2	602 216.9	22
11	Portugal	10 637 713	5	+	12.6	→	**18**	604 280.7	590 984.1	22
12	Czech Republic	10 506 813	5	+	12.4	→	**18**	602 150.0	583 711.8	22
13	Hungary	10 014 324	5	+	11.9	→	**17**	593 782.3	589 077.9	22
14	Sweden	9 340 682	5	+	11.1	→	**17**	581 352.4	549 451.9	20
15	Austria	8 375 290	5	+	9.9	→	**15**	561 221.3	558 352.7	19
16	Bulgaria	7 563 710	5	+	8.96	→	**14**	541 745.4	540 265.0	18
17	Denmark	5 534 738	5	+	6.6	→	**12**	478 876.9	461 228.2	13
18	Slovakia	5 424 925	5	+	6.4	→	**12**	474 719.8	452 077.1	13
19	Finland	5 351 427	5	+	6.3	→	**12**	471 884.1	445 952.2	13
20	Ireland	4 467 854	5	+	5.3	→	**11**	434 039.2	406 168.5	12
21	Croatia	4 425 747	5	+	5.2	→	**11**	432 042.5	402 340.6	–
22	Lithuania	3 329 039	5	+	3.9	→	**9**	372 194.3	369 893.2	12
23	Latvia	2 248 374	5	+	2.7	→	**8**	293 370.1	281 046.8	9
24	Slovenia	2 046 976	5	+	2.4	→	**8**	275 674.9	255 872.0	8
25	Estonia	1 340 127	5	+	1.6	→	**7**	203 424.7	191 446.7	6
26	Cyprus	803 147	5	+	0.95	→	**6**	134 946.5	133 857.8	6
27	Luxembourg	502 066	5	+	0.6	→	**6**	89 736.9	83 677.7	6
28	Malta	412 970	5	+	0.5	→	**6**	75 231.8	68 828.3	6
29	Iceland	317 630	5	+	0.4	→	**6**	59 079.2	52 938.3	–
	Total	*505 846 802*	*145*				***751***			*754*

Each State receives one non-base seat for every 844 000 *citizens or part thereof. Population/seat ratios are strictly decreasing before rounding, but there are three violations after rounding.*

The divisor 844 000 is such that the seat total is 751.

[21] Eurostat 1.1.2011.

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DIRECTORATE-GENERAL FOR INTERNAL POLICIES

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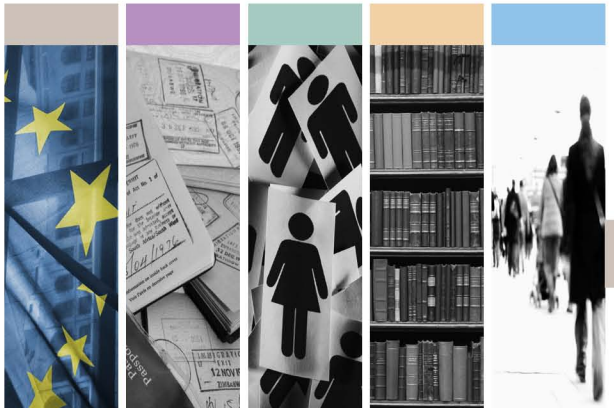
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