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Matrices and politics

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Abstract. Biproportional apportionment methods provide a novel approach of translating electoral votes into parliamentary seats. A two-way proportionality is achieved, to districts relative to their populations, and to parties relative to their total votes. The methods apply when the electoral region is subdivided into several electoral districts, each with a prespecified "district magnitude," that is, the number of seats per district. The input data thus consists of a matrix with rows and columns corresponding to districts and parties, and entries to party votes in districts. A biproportional apportionment method converts the party votes into an apportionment matrix of corresponding seat-numbers such that, within a district, the sum of the seat-numbers matches the prespecified district magnitude, while within a party, the seat-numbers sum to the overall party seats that are proportional to the vote totals across the whole electoral region. The method had its world premiere in February 2006, with the election of the Zürich City Parliament.

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1 Introduction

A new technique for converting votes into seats is described for parliamentary systems where the whole electoral region is subdivided into various electoral districts. *Biproportional apportionment methods* achieve a two-way proportionality: to the populations of the districts, and to the parties' vote totals.

In Section 2 we illustrate the approach by means of the new Zürich apportionment procedure [Neues Zürcher Zuteilungsverfahren, NZZ]. The example constitutes the world premiere of the method, the election of the Zürich City Parliament on 12 February 2006. The use of biproportional apportionment methods will undoubtedly proliferate.

The new methods may be viewed as discrete counterparts of the continuous Iterative Proportional Fitting procedure for the adjustment of statistical tables to match prespecified marginals. However, there are vital differences. Section 3 reviews the pertinent literature, contrasting the continuous and discrete aspects of the problem.

2 The new Zürich apportionment procedure

A geographical subdivision of a large electoral region into several *electoral districts* is an ubiquitous tool for ensuring that electoral systems honor historically drawn political and administrative subdivisions. Many systems apportion the total number of seats well ahead of election day, in the middle of the legislative period say, on the grounds of population counts. Thus each district is assigned its *district magnitude*, the number of seats allocated to it. The methods to carry out apportionment are well understood, as expounded in the monograph of Balinski and Young (2001).

There are and have been some ten parties in Zürich. Formerly, parties presented lists of candidates in districts, and votes were converted into seats within each of them separately. However, due to population mobility some of the districts shrank to as few as two or three seats, making it impossible to meet the ideal of proportionality. In particular, some voters could justifiably complain that their votes counted for naught, and did! A citizen in a district with few seats who repeatedly voted for a party that received no seats in the district brought suit complaining that his vote counted for nothing, and won. This provided the impetus to amend the electoral law and to implement a biproportional system.

Biproportional apportionment methods originate with Balinski and Demange (1989a,b), and were explored further by Balinski and Rachev (1993, 1997). Balinski and Ramírez-González (1997, 1999a,b) pointed out that the then Mexican electoral system suffered from severe deficiencies that might be overcome by using a biproportional method. M. B. wrote a popular science article (Balinski 2002) outlining the idea of biproportional representation and how it could answer the implicit demands of the Mexican law. F. P. translated the article into German, when shortly afterwards Christian Schuhmacher from the Zürich Justice and Interior Department hit upon the Augsburg group in the Internet. Pukelsheim and Schuhmacher (2004) adopted Balinski's idea to the Zürich situation. The *new Zürich apportionment procedure* [Neues Zürcher Zuteilungsverfahren, NZZ] celebrated its debut performance with the Zürich City Parliament election on 12 February 2006.

The 2006 Zürich election data and apportionment are presented here. Eight of the competing parties had sufficient votes to participate in the apportionment process. The initial step, the *superapportionment*, allocates all 125 City Parliament seats among the parties proportionally to their vote totals in all districts, resulting in the *overall party seats*. The superapportionment

Table 1. Biproportional divisor method with standard rounding, Zürich City Parliament election of 12 February 2006.

		SP	SVP	FDP	Grüne	CVP	EVP	AL	SD	*City divisor*
Support size		23180	12633	10300	7501	5418	3088	2517	1692	*530*
		Biproportional apportionment, based on party ballot counts								*District divisor*
	125	*44*	*24*	*19*	*14*	*10*	*6*	*5*	*3*	
"1+2"	*12*	28518-4	15305-2	21833-3	12401-2	7318-1	2829-0	2413-0	1651-0	*7000*
"3"	*16*	45541-7	22060-3	10450-1	17319-3	8661-1	2816-0	7418-1	3173-0	*6900*
"4+5"	*13*	26673-5	8174-2	4536-1	10221-2	4099-1	1029-0	9086-2	1406-0	*5000*
"6"	*10*	24092-4	9676-1	10919-2	8420-1	4399-1	3422-1	2304-0	1106-0	*6600*
"7+8"	*17*	61738-5	27906-2	51252-5	25486-2	14223-1	10508-1	5483-1	2454-0	*11200*
"9"	*16*	42044-6	31559-4	12060-2	9154-1	11333-1	9841-1	2465-0	5333-1	*7580*
"10"	*12*	35259-4	19557-3	15267-2	9689-1	8347-1	4690-1	2539-0	1490-0	*7800*
"11"	*19*	56547-6	40144-4	19744-2	12559-1	14762-2	11998-2	3623-1	6226-1	*9000*
"12"	*10*	13215-3	10248-3	3066-1	2187-1	4941-1	0-0	429-0	2078-1	*4000*
Party divisor		*1.006*	*1.002*	*1.01*	*0.97*	*1*	*0.88*	*0.8*	*1*	

A table entry is of the form p-s, where p is the party ballot count in the district, and s is the seat-number apportioned to that party's list in the district. The party ballot count p is divided by the associated district and party divisors, and then rounded to obtain s. In district "1+2", party SP had $p = 28518$ ballots and was awarded $s = 4$ seats, since $p/(7000 \times 1.006) = 4.05 \searrow 4$. The divisors (right and bottom, in italics) are such that the district magnitudes and the overall party seats (left and top, in italics) are met exactly. The overall party seats result from the superapportionment based on the electorate support sizes.

responds to the recent constitutional order to assure that each person's vote counts. It no longer matters whether voters cast their ballots in districts that are large or small.

A peculiar feature of the Zürich electoral law is that each voter has as many ballots as are given by the district magnitude. Thus voters in district "1+2" command 12 ballots, in district "3" they have 16, etc. The counts of the ballots provide the raw data that are returned from the polling stations, called *party ballot counts* [Parteistimmen], as shown in the body of Table 1. For the aggregation across the whole electoral region, the districtwise party ballot counts are adjusted so that every person (as opposed to every ballot) has equal weight. Party ballot counts are divided by the district magnitude and rounded, yielding the *district support size* [Distriktwählerzahl] of a party. District support sizes are taken to be integer numbers, in order to support the interpretation that they designate the number of people in the district who back the party considered. The sum of the district support sizes, the overall *support size* [Wählerzahl], is the number of persons who back the party across the whole electoral region (in this case: the City of Zürich). The transition to overall support sizes adjusts for the different number of ballots in the districts, so that each voter contributes equally to the superapportionment.

In Table 1, the SP's district support size in district "1+2" is 28518/12 = 2376.5 ↗ 2377, while in district "3" it is 45541/16 = 2846.3 ↘ 2846. The eight parties eligible to receive seats had overall support sizes of 23180 : 12633 : 10300 : 7501 : 5418 : 3088 : 2517 : 1692. Using the divisor method with standard rounding (often named after D. Webster or A. Sainte-Laguë), the superapportionment results in the overall party seats 44 : 24 : 19 : 14 : 10 : 6 : 5 : 3 (city divisor 530).

At the final step, the *biproportional divisor method with standard rounding* computes the *subapportionment*. It secures a two-way proportionality, verifying the prespecified district magnitudes as well as allocating all of the overall party seats. These restrictions form the left and top borders of Table 1, printed in italics. The body of the table displays the original party ballot counts. Two sets of divisors are needed, *district divisors* and *party divisors*, bordering Table 1 on the right and at the bottom. Every party ballot count is divided by the associated district divisor and the associated party divisor, and the resulting quotient is rounded in the standard way to obtain the seat-number. For instance, the SP in district "1+2" receives 28518/(7000 · 1.006) = 4.05 ↘ 4 seats. All party ballot counts in a given district are adjusted by the same (district) divisor, so that in effect they have simply been rescaled. Similarly, in all districts the party ballot counts of a given party are adjusted by the same (party) divisor, so they, too, are only rescaled. It may be proved that the resulting apportionment is unique (except possibly for ties).

Two-way proportionality is of interest in political systems beyond the one of Zürich. Bochsler (2005) studies its use for the election of the Swiss national parliament. Balinski (2004) discusses its application to elect France's representatives in the European Parliament. Biproportionality is a possible remedy to the corruptive effects of gerrymandering in the USA (Balinski 2006b), and in the current Italian electoral law it would remove "The Bug" described by Pennisi (2006). Legislative preparations to install a biproportional system are under way in the Faroe Islands (Zachariassen and Zachariasen 2005, 2006).

District and party divisors are the key quantities [Wahlschlüssel] of biproportional methods. They are not unique, since nothing is changed when the districts' divisors are multiplied by a scalar and the party divisors are divided by the same amount. Moreover, a slight variation does not matter as long as the resulting quotients round to the same integers. The divisors cannot be obtained from a closed formula, but must be determined algorithmically. The BAZI program, available at www.uni-augsburg.de/bazi, implements several approaches to finding them (Pukelsheim 2004, 2006). While BAZI now offers a selection of algorithms (Maier 2006), it originally started out with an Alternating Scaling algorithm that is similar to the Iterative Proportional Fitting procedure.

3 Biproportional apportionment and iterative proportional fitting

The breakthrough to a practically persuasive and theoretically convincing approach to the matrix biproportional apportionment problem is due to Balinski and coauthors (see Section 1). The starting point is an axiomatic theory of apportionment for vector problems developed by Balinski and Young (2001). A major result is that among all conceivable apportionment methods, *divisor methods* are the only acceptable ones. They are in one-to-one correspondence with rounding functions, that is, with the prescription of how to round a positive real number to one of its neighboring integers, in each closed interval $[n-1, n]$ ($n = 1, 2, \dots$) of the nonnegative half-line.

For vector problems, divisor methods determine a divisor (multiplier, scaling constant) so that when the input weights are scaled and rounded, using the rounding function that comes with the method, the resulting integers verify the prespecified side condition. The same approach works for matrix problems, except that now two *sets* of divisors are needed, row divisors and column divisors, and that an entry of the input weight matrix is scaled twice, by its row divisor and by its column divisor, before it is rounded to an integer. It is thus tempting to aim at a theory emphasizing the similarity of vector and matrix problems (Balinski 2006a).

Gaffke and Pukelsheim (2006a) formulate the matrix apportionment problem as an integer optimization problem, exhibiting the apportionment as the mode of a multinomial-type probability density function. This optimization approach is delineated already by Carnal (1993), for the specific divisor method with rounding down (T. Jefferson, V. D'Hondt, E. Hagenbach-Bischoff), referring to the electoral system for the Swiss Canton of Bern, see also Carnal and Riedwyl (1982). Once the primal optimization problem is set up, the row and column divisors then emerge as the values of the solution to an associated dual problem. This suggests a classification of algorithms as *primal algorithms*, or as *dual algorithms* (Gaffke and Pukelsheim 2006b).

From a statistical viewpoint, the biproportional apportionment problem is identical with the problem of adjusting a frequency table so as to meet prespecified row and column marginals. For a textbook example see Cochran (1977, page 124). The original paper on the statistical problem is Deming and Stephan (1940); the authors proposed what since has become known as the *Iterative Proportional Fitting* (IPF) procedure, but their convergence proof was flawed. Further research eventually established the conjectured convergence of the IPF procedure, see the encyclopedia article by Fienberg and Meyer (1983). In statistical jargon, the IPF procedure is sometimes called *raking* (Fagan and Greenberg 1987).

Besides statistics, Bacharach (1965, 1970) applies IPF to economic input-output analysis. Lamond and Stewart (1981) use it to solve transportation problems, and provide references from that field. In probability theory, the procedure has been used to convert a nonnegative matrix into a doubly stochastic matrix, by scaling rows and columns so that each of them sums to one. This problem generated a series of research papers, see Sinkhorn (1964, 1966, 1967, 1972), Sinkhorn and Knopp (1967), Marshall and Olkin (1968), Cottle, Duvali, and Zikan (1986), Khachiyan and Kalantari (1992).

However, IPF does not solve the problems of biproportional apportionment. It rescales a nonnegative matrix into another matrix with nonnegative *real* entries – not *integer* entries – that verify prespecified marginals. An iterative procedure, it stops when the side conditions are met to within a given error bound, so its solutions come with a disclaimer that, due to numerical inaccuracies, the marginal restrictions may not be met exactly, as in Bacharach (1970). The disclaimer is standard in statistical publications, when a frequency table is converted into percentages or tenths of a percent (Wainer 1998; Pukelsheim 1998). The disclaimer poses no problem as far as descriptive statistics or Bacharach's input-output analysis are concerned. It becomes problematic when stochastic matrices are generated where the probabilities must sum to one exactly, not just approximately. The disclaimer becomes definitely untenable in the context of apportioning the seats of a parliamentary body. It is unacceptable to leave a seat empty, or to create an extra seat, with the excuse that inaccuracies of the mathematical method cannot do better.

The IPF procedure is part of "continuous" mathematics, while biproportional apportionment belongs to "discrete" mathematics. Interestingly, this is an example where a presumably "soft science" such as political decision making insists on exact results, whereas a purportedly "exact science" such as calculus makes do with approximations.

A particular dual algorithm is alternating scaling (AS), a discrete variant of the (continuous) IPF procedure. While the IPF procedure is known to converge always, the AS algorithm may "stall", cycling from a solution that satisfies the row but not the column constraints to one that satisfies the column but not the row constraints, and back again. Extensive simulations suggest that this may happen only if there are sufficiently many ties in the solutions. For empirical election data, ties are extremely rare. Hence it is fair to say that the AS algorithm works fine, for all practical purposes. The reason is that empirical data usually are "well behaved" in that they are not only free of ties but determined by relatively large intervals of divisors. The BAZI program uses the AS algorithm because of its fast initial progress. The program safeguards against stalling by switching, if needed, to the "Tie-and-Transfer" algorithm of Balinski and Demange (1989b), as outlined by Maier (2006). Further algorithmic improvements are being investigated by Zachariasen (2006).

An entirely different approach to the biproportional problem has been proposed in the context of rounding census data. Called *controlled rounding* it may be interpreted as a generalization of the method of greatest remainders for vector rounding (often named after A. Hamilton, T. Hare / H. F. Niemeyer) to that of rounding matrices. Developed in a series of papers by Cox and coauthors (Cox and Ernst 1982; Causey, Cox, and Ernst 1985; Cox 1987, 2003; Cox and George 1989), it has been recommended for the Belgian electoral system (De Meur, Gassner, and Hubaut 1985; De Meur and Hubaut 1986; De Meur and Gassner 1987; Gassner 1988, 1989, 1991, 2000). It has severe drawbacks: it seriously distorts proportionality, and it lacks any axiomatic or theoretical justification.

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References

[See also the Proportional Representation literature list www.uni-augsburg.de/bazi/literature.html]

Bacharach, M. (1965). Estimating nonnegative matrices from marginal data. *International Economic Review* (*Osaka*), 6, 294–310.

Bacharach, M. (1970). *Biproportional Matrices & Input-Output Change*. Cambridge, UK: Cambridge University Press.

Balinski, M. (2002). Une "dose" de proportionnelle: le système électoral mexicain. *Pour la Science*, April 2002, 58–59. [German: Verhältniswahlrecht häppchenweise: Wahlen in Mexiko. *Spektrum der Wissenschaft*, October 2002, 72–74.]

Balinski, M. (2004). *Le suffrage universel inachevé*. Paris: Editions Belin.

Balinski, M. (2006a). Apportionment: uni- and bi-dimensional. In *Mathematics and Democracy. Recent Advances in Voting Systems and Collective Choice*. Eds. B. Simeone and F. Pukelsheim. New York.

Balinski, M. (2006b). Fair majority voting (or How to eliminate gerrymandering). *American Mathematical Monthly*, forthcoming.

Balinski, M. and Demange, G. (1989a). An axiomatic approach to proportionality between matrices. *Mathematics of Operations Research*, 14, 700–719.

Balinski, M. and Demange, G. (1989b). Algorithms for proportional matrices in reals and integers. *Mathematical Programming*, 45, 193–210.

Balinski, M. and Rachev, S. T. (1993). Rounding proportions: rules of rounding. *Numerical Functional Analysis and Optimization*, 14, 475–501

Balinski, M. and Rachev, S. T. (1997). Rounding proportions: methods of rounding. *Mathematical Scientist*, 22, 1–26

Balinski, M. and Ramírez-González, V. (1997). Mexican electoral law: 1996 version. *Electoral Studies*, 16, 329–340.

Balinski, M. and Ramírez-González, V. (1999a). Mexico's 1997 apportionment defies its electoral law. *Electoral Studies*, 18, 117–124.

Balinski, M. and Ramírez-González, V. (1999b). Parametric methods of apportionment, rounding and production. *Mathematical Social Sciences*, 37, 107–122.

Balinski, M. and Young, H. P. (2001). *Fair Representation: Meeting the Ideal of One Man, One Vote. Second Edition.* Washington, DC: Brookings Institution Press.

Bochsler D. (2005). Biproportionale Wahlverfahren für den Schweizer Nationalrat. Online publication: www.opus-bayern.de/uni-augsburg/volltexte/2005/160

Carnal, H. (1993). Mathématiques et politique. *Elemente der Mathematik*, 48, 27–32.

Carnal, H. and Riedwyl, H. (1982). Wahlkreisverbandsarithmetik. Technischer Bericht No. 8, Institut für Mathematische Statistik und Versicherungslehre, Universität Bern.

Cochran, W. G. (1977). *Sampling techniques, Third Edition.* New York: Wiley.

Causey, B. D., Cox, L. H., and Ernst, L. R. (1985). Applications of transportation theory to statistical problems. *Journal of the American Statistical Association*, 80, 903–909.

Cottle, R. W., Duvali, S. G., and Zikan, K. (1986). A Lagrangian relaxation algorithm for the constrained matrix problem. *Naval Research Logistics Quarterly*, 33, 55–76.

Cox, L. H. (1987). A constructive procedure for unbiased controlled rounding. *Journal of the American Statistical Association*, 82, 520–524.

Cox, L. H. (2003). On properties of multi-dimensional statistical tables. *Journal of Statistical Planning and Inference*, 117, 251–273.

Cox, L. H. and Ernst, L. R. (1982). Controlled rounding. *Information Systems and Operational Research*, 20, 423–432.

Cox, L. H. and George, J. A. (1989). Controlled rounding for tables with subtotals. *Annals of Operations Research*, 20, 141–157.

De Meur, G. and Gassner, M. (1987). Problems of equity in political representation: what we want and why we can't. In *The Logic of Multiparty Systems.* Ed. M. J. Holler. Dordrecht: Martinus Nijhoff Publishers. Pp. 391–404.

De Meur, G. and Hubaut, X. (1986). Fair models of political fairness. *European Journal of Political Research*, 14, 239–252.

De Meur, G., Gassner, M., and Hubaut, X. (1985). A mathematical model for political bipolarization. *European Journal of Political Research*, 13, 409–420.

Deming, W. E. and Stephan, F. F. (1940). On a least squares adjustment of a sampling frequency table when the expected marginals totals are known. *Annals of Mathematical Statistics*, 11, 427–440.

Fagan, J. T. and Greenberg, B. V. (1987). Making tables additive in the presence of zeros. *American Journal of Mathematical and Management Sciences*, 7, 359–383.

Fienberg, S. S. and Meyer, M. M. (1983). Iterative proportional fitting. In *Encyclopedia of Statistical Sciences, Volume 4.* Ed. S. Kotz. New York: Wiley. Pp. 275–279.

Gaffke, N. and Pukelsheim, F. (2006a). Divisor methods for proportional representation systems: an optimization approach to vector and matrix problems. Submitted for publication.

Gaffke, N. and Pukelsheim, F. (2006b). Separable convex minimization in bounded integers under totally unimodular linear equations. Submitted for publication.

Gassner, M. (1988). Two-dimensional rounding for a quasi-proportional representation. *European Journal of Political Economy*, 4, 529–538.

Gassner, M. (1989). An impossibility theorem for fair bidimensional representation: towards a biproportional solution. In *Mathematical Psychology in Progress.* Ed. E. E. Roskam. Berlin: Springer. Pp. 345–365.

Gassner, M. (1991). A solution for two-dimensional proportional representation. *Journal of Theoretical Politics*, 3, 321–342.

Gassner, M. (2000). *Représentations parlementaires. Méthodes mathématiques biproportionnelles de Répartition des Sièges.* Éditions de l'Université de Bruxelles.

Khachiyan, L. and Kalantari, B. (1992). Diagonal matrix scaling and linear programming. *SIAM Journal on Optimization*, 2, 668–672.

Lamond, B. and Stewart, N. F. (1981). Bregman's balancing method. *Transportation Research Part B: Methodological*, 15, 239–248.

Maier, S. (2006). Algorithms for biproportional apportionment methods. In *Mathematics and Democracy. Recent Advances in Voting Systems and Collective Choice*. Eds. B. Simeone and F. Pukelsheim. New York.

Marshall, A. W. and Olkin, I. (1968). Scaling of matrices to achieve specified row and column sums. *Numerische Mathematik*, 12, 83–90.

Pennisi, A. (2006). When one plus one is not two: a flawed procedure for bi-proportional allocation in Italy. In *Mathematics and Democracy. Recent Advances in Voting Systems and Collective Choice*. Eds. B. Simeone and F. Pukelsheim. New York.

Pukelsheim, F. (1998). Rounding tables on my bicycle. *Chance* 11, 57–58.

Pukelsheim, F. (2004). BAZI: a Java programm for proportional representation. *Oberwolfach Reports*, 1, 735–737.

Pukelsheim, F. (2006). Current issues of apportionment methods. In *Mathematics and Democracy. Recent Advances in Voting Systems and Collective Choice*. Eds. B. Simeone and F. Pukelsheim. New York.

Pukelsheim, F. and Schuhmacher, C. (2004). Das neue Zürcher Zuteilungsverfahren für Parlamentswahlen. *Aktuelle Juristische Praxis – Pratique Juridique Actuelle*, 5, 505–522.

Sinkhorn, R. (1964). A relationship between arbitrary positive matrices and doubly stochastic matrices. *Annals of Mathematical Statistics*, 35, 876–879.

Sinkhorn, R. (1966). A relationship between arbitrary positive matrices and stochastic matrices. *Canadian Journal of Mathematics*, 18, 303–306.

Sinkhorn, R. (1967). Diagonal equivalence of matrices with prescribed row and column sums. *American Mathematical Monthly*, 74, 402–405.

Sinkhorn, R. (1972). Continuous dependence on A in the D_1AD_2 theorems. *Proceedings of the American Mathematical Society*, 32, 395–398.

Sinkhorn, R. and Knopp, P. (1967). Concerning nonnegative matrices and doubly stochastic matrices. *Pacific Journal of Mathematics*, 21, 343–348.

Wainer, H. (1998). Visual revelations: rounding tables. *Chance*, 11, 46–50.

Zachariasen, M. (2006). Algorithmic aspects of divisor-based biproportional rounding. Typescript.

Zachariassen, P. and Zachariasen, M. (2005). Frá atkvøðum til tingsessir. Samanbering av hættum til útrokning av tingmannabýti [Appendix: Seat distributions in the Faroe Parliament 1978–2004 according to 8 electoral formulae]. University of the Faroe Islands, Faculty of Science and Technology, Technical Report 2005:1.

Zachariassen, P. and Zachariasen, M. (2006). A comparison of electoral formulae for the Faroe Parliament (The Løgting). Typescript.

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