Current Issues of Apportionment Methods

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Abstract   Three apportionment problems are addressed that are of current interest in Germany and Switzerland: the assignment of committee seats in a way that preserves the parliamentary majority-minority relation, the introduction of minimum restrictions in a two-ballot system to accomodate the direct seats won by the constituency ballots, and biproportional apportionment methods for systems with multiple districts so as to achieve proportionality between party votes as well as between district populations.

Keywords: Gentle majority clause; direct-seat restricted divisor methods; biproportional divisor methods; BAZI computer software.

1.   Introduction

Three proportional representation problems are sketched that are of practical and current interest. The first problem is to map a majority of votes into a majority of seats, encountered when the German Bundestag had to apportion sixteen committee seats. All of the methods that the Bundestag had been using so far produced a tie, assigning eight seats to the government majority and another eight to the opposition minority. A *gentle majority clause* is suggested to resolve the tie (Section 2).

The second problem concerns the election of the Bundestag deputies proper. The German Federal Electoral Law provides each voter with two ballots, a party ballot and a constituency ballot. The party ballots form the basis for a proportional apportionment of all Bundestag seats, while the constituency ballots are instrumental in identifying direct-seat winners in single-member constituencies. The Electoral Law desires to combine the two components, but actually fails to do so when setting up the operational instructions to evaluate the two ballots. Defects may evolve, the most serious – and actually fatal, in our view – defect being that more party ballots may actually cause a loss of seats. The system may thus discourage voters to cast their ballots in favor of the party of their choice! Luckily, the apportionment theory of Balinski/Young (2001) offers a remedy, by imposing minimum restrictions. *Direct-seat re-*

stricted methods evade the defects, and successfully combine the two components of the German system, of a proportional apportionment via party ballots, and of an election of persons via constituency ballots (Section 3).

The third problem considers electoral systems where the whole electoral region is subdivided into various electoral districts. We review recent work on *biproportional methods* tailored to achieve two-way proportionality, that is, proportionality among the vote counts for parties, and proportionality among the populations numbers for districts (Section 4).

2.   A Gentle Majority Clause

With the start of a legislative period, a new German Parliament [Bundestag] elects its delegates for the Bundestag-Bundesrat Conference Committee [Vermittlungsausschuss]. The Bundesrat is the assembly of the 16 states [Länder], each sending one representative into the Conference Committee. In order to be on par with the Bundesrat, the Bundestag occupies another 16 seats, apportioning them to the parliamentary factions proportional to their size. The *faction size* [Fraktiongröße] is the number of deputies belonging to the faction. In the 2002 legislative period, there were four factions, SPD, CDU/CSU, Bündnis 90/Die Grünen, and FDP, of sizes $249 : 247 : 55 : 47$.

Over the years the Bundestag has familiarized itself with three apportionment methods: the divisor method with standard rounding (Webster/Sainte-Laguë/Schepers), the divisor method with rounding down (Jefferson/D'Hondt/ Hagenbach-Bischoff), and the quota method with residual fit by largest remainders (Hamilton/Hare/Niemeyer). All of these methods allocate the 16 seat Bundestag delegation as $7 : 7 : 1 : 1$, entailing a tie of $8 : 8$ seats between the government majority (Social Democrats and Greens, $249 + 55 = 304$ seats), and the opposition minority (Conservatives and Liberals, $247 + 47 = 294$ seats).

To break the tie, the Bundestag majority passed a motion to proportionally apportion just 15 seats, and to directly assign the last seat to the largest faction. The resulting allocation $8 : 6 : 1 : 1$ secured a committee majority of $9 : 7$ for the government parties. Not surprisingly, the opposition minority challenged the apportionment in court. On 8 December 2004 the German Federal Constitutional Court ordered the Bundestag to reconsider the apportionment, but was otherwise vague and nebulous which constitutional principles the Bundestag was to observe when renewing its deliberations. The Court specified, though, that the procedure used ought to be "transparent, calculable, and abstract-general".

On 17 February 2005 the Bundestag Rules Committee, who was in charge of the proceedings, conducted an expert hearing. The opinion presented by us is published in Pukelsheim/Maier (2005). Our preferred option is a *gentle major-*

ity clause, consisting of two parts. First and foremost, the Bundestag attempts to select a committee size for which the divisor method with standard rounding (Webster/Sainte-Laguë/Schepers) yields an apportionment that preserves the majority-minority relation. The idea is not at all new, just codifying what already now is standard Bundestag practice. The first part, though, does not resolve the Conference Committee issue. The size of the Bundestag delegation is fixed at 16, the number of states in the federation. To evade the threatening tie, the divisor method with standard rounding (Webster/Sainte-Laguë/Schepers) needs to be amended.

The second part of the gentle majority clause comes to bear only in such cases when the first part results in a tie. Then the smallest possible majority in the committee is allocated with the government majority, thus leaving the largest possible committee minority for the opposition minority. Within each of the two groups, the seats available are apportioned using the divisor method with standard rounding (Webster/Sainte-Laguë/Schepers). For the Conference Committee, the government majority shares 9 seats in the relation $7 : 2$, while the opposition minority allocates the remaining 7 seats as $6 : 1$. In summary, the resulting apportionment is $7 : 6 : 2 : 1$. Pukelsheim/Maier (2005) argue that the gentle majority qualifies to be transparent, calculable, and abstract-general.

3.   Direct-Seat Restricted Methods

The German Federal Electoral Law provides every voter with two ballots, a *constituency ballot* [Erststimme] and a *party ballot* [Zweitstimme]. Voters mark the two ballots on a single sheet of paper where the choices for the constituency ballot are printed on the left half of the page, while the party ballot choices occupy the right half. To aid voters in distinguishing between the two halves, one is printed in blue, the other, in black.

The party ballots are the basis for the *superapportionment* [Oberzuteilung], a proportional apportionment of all 598 Bundestag seats among parties. Parties participate in the apportionment process only if they gain at least five percent of the valid party ballots. Thus the party ballots serve to run a proportional representation system with a five percent threshold, straight and simple. The system becomes more demanding when deciding who is going to fill the seats. The Law stipulates that the seats of a party are manned primarily by such candidates who, in their constituencies, won a relative majority of the constituency ballots. In other words, the objective of the constituency ballots is "to elect persons" in single-member districts. There are 299 constituencies, and hence the same number of winners of *direct seats* [Direktmandate].

The remaining 299 seats are filled with candidates from party lists. This is where the Law becomes tricky: party lists are organized by states, whence a

party generally commands 16 *state lists* [Landeslisten]. Of course, the idea is that deputies have roots in the geographical region where they are elected, if not in their constituency, than at least in their state. Thus the seats that a party received in the superapportionment are proportionally broken down to its 16 state lists. Hopefully the *subapportionment* [Unterzuteilung] allocates enough seats to a party in a state, to accomodate all direct-seat winners (of that party in that state). Any additional seats are filled from the state list, usually referred to as *list seats* [Listenmandate].

There remain "exceptional" cases where a party wins more direct seats in a state than the state list receives in the subapportionment. In such cases, the direct seats stay with the party, even though the proportional allocation via super- and subapportionments does not justify that many seats. This generates additional seats, called *overhang seats* [Überhangmandate], enlarging the size of the Bundestag beyond the initial 598. While the literature sometimes speaks of "surplus seats", we stick to the experts' terminolgy deliberately coined when New Zealand adopted the German electoral system (New Zealand Electoral Commission 1986). The current Bundestag comprises 614 deputies, with 9 overhang seats for the Social Democrats and 7 for the Conservatives. Alas, the 2005 election is an "exceptional" case.

Well, since 1980 *every* Bundestag has had its overhang seats. We are using quotation marks because the "exceptional" cases occur regularly. Over the years there have been 73 overhang seats (Fehndrich 2005), of which 65 benefitted the government majority no matter whether the parties composing the majority were center, left, or right. Thus the Law grants a ninety percent chance that overhang seats boost the government majority, rather than being "misplaced" with the opposition minority.

Whoever forms the majority, it is not opportune for them to question a twist in the rules instrumental to bring them into being. The 1994 Bundestag elected Helmut Kohl Chancellor with the narrowest possible margin of one vote, his government majority providing a happy home to 12 overhang seats. Who would expect an overhang chancellor to bite the hand that voted him to power? The system defies not so much the politicians who, after all, must make the best out of a parliament as is. The challenge is up to the voters, to fight for their right to electoral equality, and to the courts, to check upon the justifiability with constitutional principles.

In essence, the malalignment of party and constituency ballots causes three defects (Pukelsheim 2000, Section II). One is overhang seats. The second is *doubly successful* votes, where the constituency ballot helps electing a deputy by circumventing her or his party's state list (because the party fails to pass the five percent hurdle, or the candidate is independent), while the party ballot still enters into the aggregation of another party list. In 2002, there were at

least 270 162 voters who enjoyed the good fortune of being doubly successful (Pukelsheim 2004, page 407).

The third defect, *negative ballot weights*, is prone to prove fatal, or so we believe: more party ballots may be the cause for a party to lose a seat. The system gives rise to situations where voters are discouraged to cast their party ballots for the party of their choice!

Negative ballot weights were discussed first by Meyer (1994, page 321). The problem received some subdued press coverage, with the upshot that the electoral system entertains its oddities. Then, in the 2005 election, the defect hit all German newspapers, irritating the electorate and ridiculing the system. In the Dresden I constituency, a candidate had died shortly before the election day of September 18. This caused a shift of the election, in this constituency, to a by-election [Nachwahl] on October 2. In the main election, on September 18, the Conservatives gained four overhang seats in Sachsen state. The by-election threatened to return "too many" party votes for them, letting their proportional share grow enough to convert an overhang seat into a proportionally justified seat. The bottom line would have been the loss of one seat. The numbers speak for themselves: The Conservative voters understood, and deprived the CDU of their party ballots (Cantow/Fehndrich/Zicht 2005). The feared loss of a seat did not materialize.

Under a constitution that builds on a strict separation of powers, such as the German *Fundamental Law* [Grundgesetz], the constitutionaliy of a law is examined by the courts. The Federal Electoral Law falls under the jurisdiction of the Federal Constitutional Court. The issue of negative ballot weights was presented to the Court; surprisingly, the Court remained silent about it. With the data from the 2005 Dresden I by-election, the Court will get a chance to reconsider. The Court has otherwise upheld the Electoral Law, ruling that its commendable effort to combine the elections of persons with a proportional representation system entails the disputed defects as "necessary consequences". Here errs the Court. The defects cannot be justified as being *necessary*, in the accepted sense of the word, other than that they are consequences of the instructions in the *current* Law. There are methods evading the defects and, at the same time, coming closer to merging the two electoral principles, of electing persons and of mirroring party strenghts.

Table 1 illustrates a defect-free method, for the 2005 Bundestag election data (Schorn/Schwartzenberg 2005). The procedure is called the *direct-seat restricted divisor method with standard rounding*, and works as follows. The number d of direct seats won by a party is imposed as a minimum restriction, to make sure that enough seats are allocated to provide every constituency winner with a seat. To calculate the number of proportionally justified seats, p, the divisor method with standard rouding (Webster/Sainte-Laguë/Schepers) is used. The method divides the number of party ballots by the divisor given

in Table 1, and rounds the resulting quotient in a standard fashion (down if the fractional part is below one half, and up if it is above) to obtain p. The larger of the two numbers, denoted by $d \vee p$ (read: the larger value of d or p), is the number of seats allocated. The divisors in Table 1 are such that the seats apportioned exhaust the seats available.

For example, in the superapportionment the 16 194 665 party ballots of the SPD are divided by 76 000. The resulting quotient 213.1 is rounded to $p = 213$. Since this exceeds the number of direct seats, $d = 145$, the SPD is eligible to 213 seats, on the federal level. In the subapportionment, the 213 seats are broken down to the 16 SPD state lists. The divisor used is 80 000, shown at the bottom of the column. The SPD in Sachsen-Anhalt (ST) won $d = 10$ direct seats, but received just $p = 6$ proportionally justified seats. Formerly, the difference would have generated four overhang seats. With the direct-seat restricted method, the larger of the two numbers applies, 10. For the SPD, the direct seat component dominates in five states (HH, BB, ST, TH, SL), in two states the tally is balanced (MV, HB), and in the other nine the proportionally justified seats are effective.

4.   Biproportional Methods

The subdivision of a single large electoral region into various smaller *electoral districts* is an ubiquitous topic. The German Electoral Law, dealing with sixteen states, provides just one way of handling the issue. Another well-established approach allocates the total number of seats to the electoral districts proportionally to population counts, some time during the legislative period. With the seat numbers for each district prespecified, the votes are then evaluated separately in each district. This is the system that was in use in the Canton of Zurich, Switzerland. Due to population mobility, however, some districts shrunk to as few seats as two, in the presence of some seven and more parties competing. Naturally, the idea of proportionality must fail when apportioning just two seats among many competitors. This provided the motivation to switch to a *biproportional method*.

Biproportional apportionment methods were introduced into the literature by Balinski/Demange (1989a,b). Balinski (2002) applied the method to Mexico, in a popular science article that I translated into German. Shortly afterwards Christian Schuhmacher, from the Zurich Justice and Interior Department, hit upon the Augsburg Bazi group in the Internet. Together, we adopted Balinski's idea to the Zurich situation (Pukelsheim/Schuhmacher 2004). The *new Zurich apportionment procedure* [Neues Zürcher Zuteilungsverfahren, NZZ] had its world debut with the Zurich City Parliament election on 12 February 2006 (Balinski/Pukelsheim 2006a,b).

Table 1. Election of the sixteenth German Bundestag on 18 September 2005, direct-seat restricted divisor method with standard rounding.

	SPD	CDU	FDP	Die Linke	Grüne	CSU
	Superapportionment of the 598 Bundestag seats to parties (Divisor = 76 000)					
	16 194 665	13 136 740	4 648 144	4 118 194	3 838 326	3 494 309
	$145 \vee 213 = 213$	$106 \vee 173 = 173$	$0 \vee 61 = 61$	$3 \vee 54 = 54$	$1 \vee 51 = 51$	$44 \vee 46 = 46$
	Subapportionment of overall party seats to state lists (na = no list submitted)					
SH	655 361	624 510	173 320	78 755	144 712	na
	$5 \vee 8 = 8$	$6 \vee 8 = 8$	$0 \vee 2 = 2$	$0 \vee 1 = 1$	$0 \vee 2 = 2$	
MV	314 830	293 316	62 049	234 702	39 379	na
	$4 \vee 4 = 4$	$3 \vee 4 = 4$	$0 \vee 1 = 1$	$0 \vee 3 = 3$	$0 \vee 1 = 1$	
HH	365 546	272 418	84 593	59 463	140 751	na
	$6 \vee 5 = 6$	$0 \vee 3 = 3$	$0 \vee 1 = 1$	$0 \vee 1 = 1$	$0 \vee 2 = 2$	
NI	2 058 174	1 599 947	426 341	205 200	354 853	na
	$25 \vee 26 = 16$	$4 \vee 20 = 20$	$0 \vee 6 = 6$	$0 \vee 3 = 3$	$0 \vee 5 = 5$	
HB	155 366	82 389	29 329	30 570	51 600	na
	$2 \vee 2 = 2$	$0 \vee 1 = 1$	$0 \vee 0 = 0$	$0 \vee 0 = 0$	$0 \vee 1 = 1$	
BB	561 689	322 400	107 736	416 359	80 253	na
	$10 \vee 7 = 10$	$0 \vee 4 = 4$	$0 \vee 1 = 1$	$0 \vee 5 = 5$	$0 \vee 1 = 1$	
ST	474 909	357 663	117 155	385 422	59 146	na
	$10 \vee 6 = 10$	$0 \vee 5 = 5$	$0 \vee 2 = 2$	$0 \vee 5 = 5$	$0 \vee 1 = 1$	
BE	637 674	408 715	152 157	303 630	254 546	na
	$7 \vee 8 = 8$	$1 \vee 5 = 5$	$0 \vee 2 = 2$	$3 \vee 4 = 4$	$1 \vee 3 = 3$	
NW	4 096 112	3 524 351	1 024 924 0 13	529 967	782 551	na
	$40 \vee 51 = 51$	$24 \vee 44 = 44$	$24 \vee 44 = 44$	$0 \vee 7 = 7$	$0 \vee 10 = 10$	
SN	649 807	795 316	269 623	603 824	126 850	na
	$3 \vee 8 = 8$	$14 \vee 10 = 14$	$0 \vee 4 = 4$	$0 \vee 8 = 8$	$0 \vee 2 = 2$	
HE	1 197 762	1 131 496	392 123	178 913	340 288	na
	$13 \vee 15 = 15$	$8 \vee 14 = 14$	$0 \vee 5 = 5$	$0 \vee 2 = 2$	$0 \vee 5 = 5$	
TH	432 778	372 435	115 009	378 340	69 976	na
	$6 \vee 5 = 6$	$3 \vee 5 = 5$	$0 \vee 1 = 1$	$0 \vee 5 = 5$	$0 \vee 1 = 1$	
RP	822 074	877 632	278 945	132 154	172 900	na
	$5 \vee 10 = 10$	$10 \vee 11 = 11$	$0 \vee 4 = 4$	$0 \vee 2 = 2$	$0 \vee 2 = 2$	
BY	1 806 548	na	673 817	244 701	559 941	3 494 309
	$1 \vee 23 = 23$		$0 \vee 9 = 9$	$0 \vee 3 = 3$	$0 \vee 7 = 7$	$44 \vee 46 = 46$
BW	1 754 834	2 283 085	693 835	219 105	623 091	na
	$4 \vee 22 = 22$	$33 \vee 29 = 33$	$0 \vee 9 = 9$	$0 \vee 3 = 3$	$0 \vee 8 = 8$	
SL	211 201	191 067	47 188	117 089	37 489	na
	$4 \vee 3 = 4$	$0 \vee 2 = 2$	$0 \vee 1 = 1$	$0 \vee 2 = 2$	$0 \vee 0 = 0$	
Divisor	*80 000*	*79 300*	*77 000*	*77 000*	*75 000*	*76 000*

SH	Schleswig-Holstein	HB	Bremen	NW	Nordrhein-Westfalen	RP Rheinland-Pfalz
MV	Mecklenburg-Vorpommern	BB	Brandenburg	SN	Sachsen	BY Bayern
HH	Hamburg	ST	Sachsen-Anhalt	HE	Hessen	BW Baden-Württemberg
NI	Niedersachsen	BE	Berlin	TH	Thüringen	SL Saarland

The seats apportioned are written as $d \vee p$, that is, the larger value of d or p, where d is the count of direct seats won and p is number of proportionally justified seats. In the superapportionment, the SPD entry $145 \vee 213 = 213$ means that the party won 145 direct seats, while its party ballots justify 213 seats; the larger number prevails, 213.

Table 2. Biproportional divisor method with standard rounding, retrospectively applied to the 2002 Zurich City Parliament election.

		SP	SVP	FDP	Grüne	CVP	SenL	AL	*City divisor*
Electorate support		33287	17753	15307	8299	6072	3475	3223	*710*
		Biproportional apportionment of overall party and district lists							
	125	*47*	*25*	*22*	*12*	*9*	*5*	*5*	*District divisor*
"1+2"	*12*	42192-4	20508-2	28956-3	12960-2	7668-1	2964-0	2208-0	*9600*
"3"	*16*	68219-6	28897-3	16992-2	13752-2	8619-1	5428-1	8040-1	*10400*
"4+5"	*13*	40339-6	9854-1	7358-1	11271-2	6071-1	1781-0	12220-2	*7000*
"6"	*10*	36257-4	13491-2	14874-2	9556-1	4708-1	3592-0	2797-0	*8000*
"7+8"	*17*	84456-5	41191-2	74018-5	32963-2	16456-1	8245-1	6987-1	*16480*
"9"	*16*	58119-6	43585-5	20258-2	11681-1	15130-1	7717-1	3684-0	*9500*
"10"	*12*	49241-5	25620-2	24797-3	10621-1	7762-1	5351-0	4355-0	*10706*
"11"	*19*	77998-7	63333-5	30541-3	14643-1	18027-1	12088-1	4685-1	*11515.5*
"12"	*10*	19700-4	15159-3	4861-1	2105-0	4462-1	3438-1	650-0	*5000*
Party divisor		*1.022*	*1*	*0.9*	*0.87*	*1.08*	*1*	*0.81366*	

The table entries *p-s* list party votes p and seat numbers s. To obtain s, party votes p are divided by the associated district and party divisors, and then rounded. In District "1+2", party SP wins $p = 42192$ votes and gets $s = 4$ seats, since $p/(9600 \times 1.022) = 4.3 \searrow 4$. The divisors (right and bottom, in italics) are such that the prespecified district seats and the overall party seats (left and top, in italics) are met exactly. The overall party seats result from the superapportionment, on the basis of electorate supports.

Table 2 shows the method at work in a hypothetical, restrospective evaluation of the past 2002 election data. In order to participate in the apportionment process, the five percent threshold must be passed in at least one district. In 2002, this would have left seven parties. The first step then is the *superapportionment*, the apportionment of all 125 parliament seats among parties, proportionally to their electorate support. This step responds to the constitutional demand that all voters contribute to the electoral outcome equally. Other than with the former system of separate district evaluations, it no longer matters whether voters cast their ballots in districts that are large or small. The second step is the *subapportionment*: The overall party seats are handed down to the districts, while verifying the prespecified district totals. Mathematics guarantees that, when a biproportional method is used, the resulting apportionment is unique (up to ties).

A complication arises since a Zurich voter is provided with as many ballots as the district has seats to fill. Thus voters in District "1+2" command 12 ballots, in District "3" they have 16, etc. The ballots may be split among parties, and cumulated. The resulting counts are called *party ballots* [Parteistimmen]; these are the raw data returned from the polling stations. The districtwise party ballots need to be aggregated across the whole electoral region. To this end, party ballots are divided by the district magnitude and rounded, yielding the *district support* [Wahlkreis-Wählerzahl] of a party. The sum of the district supports is called the *electorate support* [(Kanton-)Wählerzahl] of a party, in-

dicating how many voters back the party across the whole electoral region. Since the conversion to support quantities adjusts for the distinct number of ballots handed out in a district, every voter contributes to the superapportionment in an equal manner.

In Table 2, the SP enjoys in District "1+2" a district support of $42192/12 = 3516$, while in District "3" the support is $68219/16 = 4263.7 \nearrow 4264$. The seven parties participating in the apportionment process turn out to win electorate supports of $33287 : 17753 : 15307 : 8299 : 6072 : 3475 : 3223$. Using the divisor method with standard rounding (Webster/Sainte-Laguë/Schepers), the superapportionment allocates the 125 seats according to $47 : 25 : 22 : 12 : 9 : 5 : 5$ (city divisor 710).

The subapportionment employs the *biproportional method with standard rounding*. It achieves a two-way proportionality, while verifying the prespecified district magnitudes as well as exhausting the overall party seats just calculated. The restrictions form the left and top borders of Table 2, typeset in italics. The method aims at proportionality among the party votes that form the table body. Two sets of divisors come into play, *district divisors* and *party divisors*, bordering Table 2 on the right and at the bottom (in italics).

The method divides the party votes by the associated district and party divisors, and rounds the resulting quotient in a standard fashion to obtain the seat number. For instance, the SP in District "1+2" receives $42192/(9600 \times 1.022) = 4.3 \searrow 3$ seats. The same district divisor is used for the vote counts of all parties, in any given district, thus treating parties districtwise equally. Similarly, the same party divisor is applied to the vote counts in all districts, for any given party, again honoring the proportionality principle. The biproportional apportionment is *coherent*, in that it fairly approximates the ideal shares of seats a party may claim when contesting individual seats (Balinski/Pukelsheim 2006b).

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