LINEAR ALGEBRA AND ITS APPLICATIONS 48-57-62 (1892)

\[
\begin{pmatrix}
X \\
Y
\end{pmatrix}
= Z
\]

The distance between X and Y is minimal.

Consider two p-variate normal distributions with zero means and positive definite dispersion matrices \(\Sigma_1\) and \(\Sigma_2\). In the theory of strong approximations, we determine their correlation matrices \(\rho_1\) and \(\rho_2\). For two p-dimensional random vectors \(X\) and \(Y\) with dispersion matrices \(\Sigma_1\) and \(\Sigma_2\), respectively, we derive the correlation matrix \(\rho\) of \(X\) and \(Y\), and conditions under which this correlation matrix \(\rho\) is sufficient to determine the strong approximation.

1. Introduction

Another context for the distance between \(X\) and \(Y\) is a dual to this problem that is of interest in the L^2-distance between \(X\) and \(Y\). There is a dual to this problem that is of interest in statistics.

ABSTRACT

Submitted by Richard W. Cule

Federal Republic of Germany
D-7800 Freiburg im Breisgau
Albert-Ludwigs-Universität
Institut für Mathematik, Stochastik
F. R. Deutschland

University of California, San Diego
Department of Statistics
1000 La Jolla Village Drive
San Diego, California 92093-0112
Theorem 2.1 (Optimality Principle) 

For any feasible solution, there exists a dual variable vector such that the objective function is minimized.

\[ \min \{ c^T x : A x = b, x \geq 0 \} \]

is equivalent to the dual problem:

\[ \max \{ b^T y : y^T A = c, y \geq 0 \} \]

where:

- \( x \) is the primal variable vector
- \( y \) is the dual variable vector
- \( c \) is the cost vector
- \( A \) is the constraint matrix
- \( b \) is the right-hand side vector

The dual problem is simpler and often easier to solve than the primal problem, especially when the number of variables in the primal is significantly larger than the number of constraints.

### Dual Theorem

Consider the dual problem:

\[ \max \{ b^T y : y^T A = c, y \geq 0 \} \]

where:

- \( y \) is the dual variable vector
- \( A \) is the constraint matrix
- \( c \) is the cost vector
- \( b \) is the right-hand side vector

### Lagrange Duality

The Lagrange dual function is defined as:

\[ g(\lambda) = \inf_{x \geq 0} \{ c^T x + \lambda^T (b - Ax) \} \]

where \( \lambda \) is the Lagrange multiplier vector.

The dual problem is equivalent to:

\[ \max \{ g(\lambda) : \lambda \geq 0 \} \]

### Interior Point Method

Interior point methods are algorithms for solving linear and nonlinear optimization problems. They work by iteratively improving a feasible solution by moving closer to the optimal solution while staying strictly inside the feasible region.

### Infeasibility

If the primal problem is infeasible, then the dual problem is unbounded.

### Duality Gap

The duality gap is the difference between the optimal values of the primal and dual problems.

\[ \text{duality gap} = \text{primal optimum} - \text{dual optimum} \]

### Complementary Slackness Conditions

If the primal problem is feasible and the dual problem is feasible, then the complementary slackness conditions hold:

- For all constraints, the product of the primal variable and the dual variable is zero.
- For all variables, the product of the primal variable and the dual variable is zero.

### References

\[ \begin{align*}
\text{In general, } & \quad \mathbf{A} \mathbf{x} = \mathbf{B}, \\
\text{then the solutions } & \quad \mathbf{x} = \mathbf{A}^{-1} \mathbf{B} = (\mathbf{A}^{-1})^T \mathbf{B}. \\
\end{align*} \]

\[ \begin{align*}
\text{Remark 4 (Duality): If } \mathbf{A} \text{ has full rank, then the problems } \\
\text{are equivalent and the solutions } \mathbf{x} = \mathbf{A}^{-1} \mathbf{B}. \\
\end{align*} \]

The condition \( \mathbf{x} = \mathbf{A}^{-1} \mathbf{B} \) is necessary and sufficient when \( \mathbf{A} \) is full rank.

\[ \begin{align*}
\text{The condition } & \quad \mathbf{x} = \mathbf{A}^{-1} \mathbf{B} \text{ is necessary and sufficient when } \mathbf{A} \text{ is full rank.} \\
\end{align*} \]
which provides a more symmetric expression in $1^{11/2}$ and $\mathbf{v}^{11/2}$.

\[
(\mathbf{z}^{11/2}) = \left(\frac{x^{11/2}}{\mathbf{z}^{11/2}} \mathbf{v}^{11/2}\right) \mathbf{v}^{11/2}
\]

\[
\left[\frac{1}{\mathbf{v}^{11/2}} \left(\mathbf{z}^{11/2} \mathbf{v}^{11/2}\right) \mathbf{v}^{11/2}\right] \mathbf{v}^{11/2} = \mathbf{v}^{11/2}
\]

value becomes $\sum$ because $\mathbf{v}^{11/2}$, the characteristic roots of $\mathbf{v}$, are the optimal

\[
\mathbf{v}^{11/2} = \left[\frac{1}{\mathbf{v}^{11/2}} \left(\mathbf{z}^{11/2} \mathbf{v}^{11/2}\right) \mathbf{v}^{11/2}\right] \mathbf{v}^{11/2} = \mathbf{v}^{11/2}
\]

However, representation of its unique optimal solution is

\[
\mathbf{v}^{11/2} = \left[\frac{1}{\mathbf{v}^{11/2}} \left(\mathbf{z}^{11/2} \mathbf{v}^{11/2}\right) \mathbf{v}^{11/2}\right] \mathbf{v}^{11/2} = \mathbf{v}^{11/2}
\]

meaning that an optimal solution is

\[
\mathbf{v}^{11/2} = \left[\frac{1}{\mathbf{v}^{11/2}} \left(\mathbf{z}^{11/2} \mathbf{v}^{11/2}\right) \mathbf{v}^{11/2}\right] \mathbf{v}^{11/2} = \mathbf{v}^{11/2}
\]

and hence is also optimal

\[
\mathbf{v}^{11/2} = \left[\frac{1}{\mathbf{v}^{11/2}} \left(\mathbf{z}^{11/2} \mathbf{v}^{11/2}\right) \mathbf{v}^{11/2}\right] \mathbf{v}^{11/2} = \mathbf{v}^{11/2}
\]

Note

\[
\mathbf{v}^{11/2} = \left[\frac{1}{\mathbf{v}^{11/2}} \left(\mathbf{z}^{11/2} \mathbf{v}^{11/2}\right) \mathbf{v}^{11/2}\right] \mathbf{v}^{11/2} = \mathbf{v}^{11/2}
\]

According to Table 1, for dimensions and

\[
\mathbf{v}^{11/2} = \left[\frac{1}{\mathbf{v}^{11/2}} \left(\mathbf{z}^{11/2} \mathbf{v}^{11/2}\right) \mathbf{v}^{11/2}\right] \mathbf{v}^{11/2} = \mathbf{v}^{11/2}
\]

IEEE TRANSACTIONS ON INFORMATION THEORY

Volume 11, January 1965

REFERENCES


