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Mathematics and Democracy

Recent Advances in Voting Systems and Collective Choice

With 33 Figures and 50 Tables

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Preface

Voting Systems and Collective Choice are subjects at the crossroad of social and exact sciences. Social sciences look at the development and evolution of electoral systems and other rules of collective choice, in the context of the changing needs and political patterns of society. Exact sciences are concerned instead with the formal study of voting mechanisms and other preference aggregation procedures, under axioms which are meant to reflect such universal principles as equity, representation, stability, and consistency.

Mathematics and Democracy presents a collection of research papers focussing on quantitative aspects of electoral system theory, such as game-theoretic, decision-theoretic, statistical, probabilistic, combinatorial, geometric, and optimization-based approaches. Electronic voting protocols and other security issues have also recently been devoted a great deal of attention. Quantitative analyses provide a powerful tool to detect inconsistencies or poor performance in actual systems. The topics covered are at the forefront of the research in the field:

- Proportional methods
- Biproportional apportionment
- Approval voting
- Gerrymandering
- Metric approaches to Social Choice
- Impossibility theorems

Applications to concrete cases such as the procedures used to elect the EU Parliament, the US Congress, and various national and regional assemblies are discussed, as well as issues related to committee voting.

Mathematicians, economists, statisticians, computer scientists, engineers, and quantitatively oriented political and social scientists will find this book not only of the highest quality, but also eclectic, discussion-oriented, and mind-provoking. Altogether, *Mathematics and Democracy* offers an appraisal from leading specialists of "what's new" in the emerging multidisciplinary science of voting systems and social choice, with a broad view on real-life applications.

The present book grew out of the *International Workshop on Mathematics and Democracy: Voting Systems and Collective Choice*, which took place 18–23 September 2005 at the Ettore Majorana Centre in the charming medieval town of Erice, Sicily, under our joint scientific directorship. The Workshop's aim was to bring together different viewpoints on the subject, and to stress the role of mathematics towards a deeper understanding, a rational assessment, and a sound design of voting procedures. The invited speakers, representing many countries, are prominent scholars from their disciplines. Many of the papers included here were first presented at the Workshop, while some were adjoined later on. All papers were refereed; we would like to thank the referees for their indispensible contributions.

The idea of the Workshop goes back as far as to the year 1999. As Bruno Simeone recalls: "In the Spring 1999 I had the chance to organize a Mini-symposium on Electoral Systems at DIMACS. There I met people like Steve Brams and Don Saari, and the idea of having a workshop in Erice came up. That Summer, I was enjoying my vacations in Santa Marinella, a seaside town north of Rome, when I got a phone call from Professor Antonino Zichichi, the well-known physicist who heads the Erice Majorana Centre: *How about giving a talk in Erice on Mathematics and Democracy in front of a few Nobel Laureates?* The title of the Workshop—and of this book—was born at that very moment. Needless to say, my vacations dissolved and I gave that talk. In Erice, I proposed to Professor Zichichi the idea of a Workshop on Voting Systems and he instantly endorsed it. He envisaged in the Workshop an excellent opportunity to encourage a multi-disciplinary debate among the most qualified international experts of electoral systems and to promote exchange between the academic world and the wider society in order to disseminate scientific findings which are of collective interest, according to the well-established tradition of the Centre. However, due to the casual intertwining of many events, some years elapsed before the project actually materialized. The turning point was the Oberwolfach Workshop on electoral systems organized by Michel Balinski, Steven Brams, and Friedrich Pukelsheim. There I had the opportunity to ask many distinguished participants whether they would be interested in a second Workshop to take place in Erice. Their massive favourable reaction convinced me to go ahead and do it."

Drawing up the conclusions of the Workshop and relying on input from several invited speakers, Professor Michel Balinski wrote down a "Declaration" for the proper choice of an electoral system. This document, now better known as The *Erice Decalogue* and already translated in other languages, was unanimously undersigned by the participants. We are particularly pleased to include in this volume (pages xi–xii) such an authoritative document providing sound guidelines for electoral reform planning and advocating quantitative methods for electoral system assessment.

Neither the Workshop nor the book would have been possible without the intervention of many persons and institutions. We are most grateful to Professor Antonino Zichichi, President of the *Ettore Majorana Foundation and Centre for Scientific Culture*, for his enthusiastic support and encouragement. We acknowledge the indispensable financial and logistic support of the Centre and its staff, headed by Dr. Fiorella Ruggiu.

We are grateful to Professor Renato Guarini, Rector of La Sapienza University; Professor Gabriella Salinetti, Dean of the Faculty of Statistical Sciences; and to Professor Paolo Dell'Olmo, Head of the Statistics Department, for their patronage and financial help. Additional funding was made available by the Italian Ministry of University and Research, and by the Sicily Region.

We are much indebted to Professor Maurice Salles and Dr. Martina Bihn of Springer-Verlag, for their prompt endorsement of our proposal to publish this book in the Springer series *Studies in Choice and Welfare*.

Our warmest thanks to the other two formidable members of the Organizing Committee, Federica Ricca and Aline Pennisi, for their precious, ubiquitous, and clever assistance in the Workshop organization and logistics. They were efficiently backed by Isabella Lari and Andrea Scozzari, to whom we extend our thanks. Sebastian Maier and Federica Ricca have done a fantastic and supersonic job in editing the LaTeX format of the articles for the book.

BRUNO SIMEONE AND FRIEDRICH PUKELSHEIM

Rome and Augsburg, July 2006

Contents

Preface v

The Erice Decalogue xi

Power Indices Taking into Account Agents' Preferences 1
Fuad Aleskerov

The Sunfish Against the Octopus: Opposing Compactness to Gerrymandering 19
Nicola Apollonio, Ronald I. Becker, Isabella Lari, Federica Ricca, Bruno Simeone

Apportionment: Uni- and Bi-Dimensional 43
Michel Balinski

Minimum Total Deviation Apportionments 55
Paul H. Edelman

Comparison of Electoral Systems: Simulative and Game Theoretic Approaches 65
Vito Fragnelli, Guido Ortona

How to Elect a Representative Committee Using Approval Balloting 83
D. Marc Kilgour, Steven J. Brams, M. Remzi Sanver

On Some Distance Aspects in Social Choice Theory 97
Christian Klamler

Algorithms for Biproportional Apportionment 105
Sebastian Maier

Distance from Consensus: A Theme and Variations 117
Tommi Meskanen, Hannu Nurmi

A Strategic Problem in Approval Voting 133
Jack H. Nagel

The Italian Bug: A Flawed Procedure for Bi-Proportional Seat Allocation 151
Aline Pennisi

Current Issues of Apportionment Methods 167
Friedrich Pukelsheim

A Gentle Majority Clause for the Apportionment of Committee Seats 177
Friedrich Pukelsheim, Sebastian Maier

Allotment According to Preferential Vote: Ecuador's Elections 189
Victoriano Ramírez

Degressively Proportional Methods for the Allotment of the European Parliament Seats Amongst the EU Member States 205
Victoriano Ramírez, Antonio Palomares, Maria L. Márquez

Hidden Mathematical Structures of Voting 221
Donald G. Saari

A Comparison of Electoral Formulae for the Faroese Parliament 235
Petur Zachariassen, Martin Zachariassen

List of Talks 253

List of Participants 255

The Erice Decalogue

The objective of the scientific research presented and discussed at the International Workshop on *Mathematics and Democracy: Voting Systems and Collective Choice* is the development and understanding of fair electoral systems. Pursued by scientists coming from different disciplines – mathematics, political science, economics, the law, computer science, ... – there is a common set of principles that the participants share.

We believe that a fair electoral system for electing a Parliament should:

1 Ensure transparency and simplicity. Voting systems whose properties are simple to understand by the electorate should be preferred to complex ones, and they should respect a nation's historical and legal context.

2 Guarantee accuracy. The act of voting – with paper ballots, optical scanners, electronic or other devices – should be able to assure to voters that their votes were accurately counted.

3 Promote competitiveness and avoid partisan bias. The system should favor no political group over another. In particular, it should render (almost) impossible the election of a majority in the Parliament with a minority of the voters.

4 Make every vote count. A system should never discourage a citizen from voting; it must encourage participation.

5 Make the Parliament a "mirror" of the electorate representing the divergent "popular wills," yet capable of governing (through, for example, the emergence of a majority).

6 Minimize the incentives to vote strategically. The system should encourage voters to express sincerely their true preferences.

7 Eliminate partisan political control by assigning the legal and administrative responsibility for elections to an independent commission.

A system using electoral districts should:

8 Encourage geographical compactness of the districts and respect natural geographical features and barriers.

9 Respect existing political subdivisions and communities of interest, and make every effort to avoid confusion among districts defined for different elections (local, regional and national).

10 Guarantee redistricting on a regular basis to account for demographic changes (but never in response to partisan appetites); at same time, it should recognize the limited precision and transitory nature of census data.

Theory is necessary to understand the properties and consequences of choosing one or another electoral system. A "science" of electoral systems is emerging and should be used in designing new systems or reforming old ones. Regrettably, history demonstrates that elected officials have repeatedly manipulated systems for partisan advantage ... and have resisted the "intrusions" of scientific approaches to the design of electoral systems. Few voters realize the extent to which manipulation has profoundly effected electoral outcomes, sometimes transforming the votes of a minority into a majority in Parliament.

All too often the players of a nation's political game – its elected officials – are, at one and the same time, the referees of the game, and they change the rules to accommodate new situations. Imagine the public outcry were the game to be football! This is why independent commissions are needed, together with professionals trained in the emerging multi-disciplinary science of electoral systems, responsible for keeping abreast of all the new theoretical and technological developments in voting.

Erice, 23 September 2005

Editors' note: The present declaration, now known under the name of "The Erice Decalogue", has been unanimously signed by all the participants in the Erice Workshop. It has been written by Professor Michel Balinski with contributions by several invited speakers and the editorial assistance of Dr. Isabella Lari.