Proportional Representation

Friedrich Pukelsheim

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Apportionment Methods and Their Applications

Second Edition

With a Foreword by Andrew Duff MEP

Springer

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ISBN 978-3-319-64706-7 ISBN 978-3-319-64707-4 (eBook)
DOI 10.1007/978-3-319-64707-4

Library of Congress Control Number: 2017954314

Mathematics Subject Classification (2010): 91B12

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

The just, then, is a species of the proportionate . . .. For proportion is equality of ratios, and involves four terms at least . . .; and the just, too, involves at least four terms, and the ratio between one pair is the same as that between the other pair; for there is a similar distinction between the persons and between the things. As the term A*, then, is to* B*, so will* C *be to* D*, and therefore,* alternando*, as* A *is to* C*,* B *will be to* D . . ..

This, then, is what the just is—the proportional; the unjust is what violates the proportion. Hence one term becomes too great, the other too small, as indeed happens in practice; for the man who acts unjustly has too much, and the man who is unjustly treated too little, of what is good . . ..

This, then, is one species of the just.

Aristotle*, Nichomachean Ethics, Book V, Chapter 3.*
Translated and Introduced by Sir David Ross, 1953.
The World's Classics 546, Oxford University Press.

Foreword to the First Edition

The virtue of parliamentary democracy rests on the representative capability of its institutions. Even mature democratic states cannot take the strength of its representative institutions for granted. Newer democracies seek practicable ways and means on which to build lasting structures of governance which will command the affinity of the people they are set up to serve. The debate about the structural reform of parliamentary democracies is never far away. Nor should it be. The powers and composition of parliamentary chambers, their rules and working methods, the organization and direction of the political parties which compete for votes and seats, the electoral systems (who to register, how to vote, how to count), and the size and shape of constituencies—all these and more are rightly subject to continual appraisal and are liable to be reformed.

Electoral reform is a delicate business: handled well, it can be the basis on which new liberal democracies spread their wings; it can refresh the old, tired democracies. Handled badly, electoral reform can distort the people's will, entrench the abuse of power, and sow the seeds of destruction of liberty. Electoral systems are central to the debate in emerging democracies, and the relatively new practice of election observation by third parties highlights the need for elections to be run not only fairly but also transparently. Voting and counting should be simple, comprehensible, and open to scrutiny—qualities which are too often lacking even in old established democracies.

Electoral reform is also very difficult to achieve. Those who must legislate for it are those very same people who have a vested interest in the status quo. That *turkeys don't vote for Christmas* is amply demonstrated in the UK, where reform of the House of Lords has been a lost cause for over a century. Advocates of reform need to stack up their arguments well, be persistent, and enjoy long lives.

Friedrich Pukelsheim has written a definitive work on electoral reform. He takes as his starting point the simple premise that seats won in a parliamentary chamber must represent as closely as possible the balance of the votes cast in the ballot box. Rigorous in his methodology, the author knows that there is no single perfect electoral system: indeed, in their quirky details, every system affects the exact outcome of an election. We are fortunate indeed that this professor of mathematics is a profound democrat. He ably brings to the service of politicians the science of the mathematician.

Dr Pukelsheim was an indispensable participant at the meeting in Cambridge in 2011, chaired by Geoffrey Grimmett, which devised "CamCom"—the best consensual solution to the problem of how to apportion seats in the European Parliament. As the Parliament's rapporteur for electoral procedure, I am happy that our ideas are now taken forward in this publication.

The European Parliament

The European Parliament presents unusual challenges both to the scientist and practitioner. It is one chamber of the legislature of the European Union with a lot of power but little recognition. It reflects a giant historical compromise between the international law principle of the equality of states and the democratic motto of "One person, one vote."

Proportional representation at the EU level needs to bear in mind not only party but also nationality. The European Parliament is the forum of the political single market where the different political cultures and constitutional practices of the 28 member states meet up. MEPs are constitutionally *representatives of the Union's citizens*, but they are elected not by a uniform electoral procedure but by different procedures under which separate national political parties and candidates fight it out, largely untroubled by their formal affiliation to European political parties.[1] Efforts to make more uniform the election of the world's first multinational parliament to be directly elected by universal suffrage have been frustrated.

Voter turnout, as we know, has declined at each election to the European Parliament from 62% in 1979 to 43% in 2009, although these overall figures disguise sharp contrasts among the states and between elections. The long financial and economic crisis since 2008 has brought to a head a crisis of legitimacy for the European Parliament. If the euro is to be salvaged, and the EU as a whole is to emerge strengthened from its time of trial, transnational democracy needs to work better. Banking union and fiscal union need the installation of federal government. That federal government must be fully accountable to a parliament which connects directly to the citizen and with which the citizen identifies. The parliament must be composed in a fair and logical way best achieved in accordance with a settled arithmetical formula and not as a result of unseemly political bartering which borders on gerrymandering and sparks controversy.

It is probable that in spring 2015 there will be a new round of EU constitutional change. This will take the form of a Convention in which heads of government and the European Commission will talk things through with members of the European and national parliaments. Part of the complex negotiations must include the electoral

[1] Article 14(2), Treaty on European Union.

reform of the European Parliament. This will be the chance to progress CamCom for the apportionment of state seats alongside an ambitious proposal for the creation of a pan-European constituency for which a certain number of MEPs would be elected from transnational party lists.[2]

There is no reason to doubt that the notion of *degressive proportionality*, which strikes mathematicians as odd, will survive these negotiations because it expresses quite well the broadly understood belief that in a federal polity, the smaller need to be protected from subordination to the larger. CamCom copes logically with degressive proportionality in a way which should satisfy even the austere requirements of the Bundesverfassungsgericht at Karlsruhe.

Nevertheless, as Friedrich Pukelsheim recognizes, fully fledged CamCom means radical adjustments to the number of MEPs elected in several states. It is important, therefore, that changes to the electoral system for one chamber of the legislature are balanced by changes to the electoral system in the other. Here, the Jagiellonian Compromise, which uses the square root as the basis for weighing the votes of the member states in the Council, deserves a good hearing.

In June 2013, the Council and European Parliament eventually agreed that the new member state of Croatia should have 11 MEPs in the Parliament which were elected in May 2014. We worked hard to ensure that the reapportionment of seats would not contradict the logic of CamCom. There is a first, albeit clumsy, legal definition of degressive proportionality. More importantly, the European Union has now formally decided to pursue the objective of a formulaic approach to the future distribution of seats in the Parliament, coupled with a commitment to revisit the matter of qualified majority voting (QMV) in the Council.

The decision of the European Council, now agreed by the European Parliament, lays down that a new system will be agreed in good time before the 2019 elections which *in the future will make it possible, before each fresh election to the European Parliament, to allocate the seats between Member States in an objective, fair, durable, and transparent way, translating the principle of degressive proportionality as laid down in Article 1, taking account of any change in their number and demographic trends in their population, as duly ascertained thus respecting the overall balance of the institutional system as laid down in the Treaties*.

So perhaps CamCom and JagCom are destined to surface together in the next EU treaty. Legislators who care to understand the maths should start with this book.

Cambridge, UK
September 2013

Andrew Duff MEP

[2]For a full exposition of this proposal, see Spinelli Group, *A Fundamental Law of the European Union*, Bertelsmann Stiftung 2013.

Preface to the Second Edition

In this second edition, the text of the first edition has been completely revised and expanded. Many empirical election results that serve as examples have been updated. Four changes deserve particular mention.

Chapter 1 contains a review of the European Parliament elections of 2014.

Chapter 9 has been enlarged by a novel and stringent proof of the Coherence Theorem of Balinski and Young.

The presentation of double-proportional divisor methods, formerly condensed into a single chapter, now extends over two chapters. Chapter 14 illustrates the practice of double proportionality, while Chap. 15 explains the required theory.

Chapter 16 is entirely new. It assembles biographical sketches and authoritative quotes from individuals who coined the development of apportionment methodology.

The author is grateful to the attentive readers of the first edition who provided feedback and valuable criticisms.

Particular thanks are due to a team of expert scholars and devoted friends who worked through most or all of the draft for this second edition. Their critical advice and numerous suggestions were vital to improve and clarify the exposition. I would like to thank *Wolfgang Bischof*, Hochschule Rosenheim (DE); *Xavier Mora*, Universitat Autònoma de Barcelona (ES); *Grégoire Nicollier*, Haute Ecole d'Ingénierie, Sion (CH); *Antonio Palomares*, Universidad de Granada (ES); and *Ben Torsney*, University of Glasgow (UK).

Augsburg, Germany — Friedrich Pukelsheim
June 2017

Preface to the First Edition

Proportional representation systems determine how the political views of individual citizens, who are many, mandate the Members of Parliament, who are but a few. The same techniques apply when in the Parliament, the political groups are to be represented in a committee of a size much smaller than the Parliament itself. There are many similar examples all showing that proportional representation inevitably culminates in the task of translating numbers into numbers—large numbers of those to be represented into small numbers of those serving as representatives. The task is solved by procedures called apportionment methods. Apportionment methods and their applications are the theme of this work. A more detailed *Outline of the Book* follows the *Table of Contents*.

By profession a mathematician rather than a politician, I have had the privilege of getting involved in several proportional representation reform projects in recent years. These include the introduction of a double-proportional electoral system in several Swiss cantons since 2006, the amendment of the German Federal Election Law during 2008–2013, and the discussion of the future composition of the European Parliament. The practical challenges and the teaching experience of many lectures and seminars on the subject of proportional representation and apportionment methods have shaped my view and provided the basis for this book.

Apportionment methods may become quite complex. However, these complexities are no ends in themselves. They are reflections of the historical past of a society, its constitutional framework, its political culture, its identity. On occasion, the complexities are due to partisan interests of the legislators responsible. This mélange turns the topic into a truly interdisciplinary project. It draws on such fields as constitutional law, European law, political sciences, medieval history, modern history, discrete mathematics, stochastics, and computational algorithms, to name but a few. I became increasingly fascinated by the interaction of so many disciplines. My fascination grew when I had the pleasure of conducting student seminars jointly with colleagues from the humanities on topics of common interest. These experiences made me realize that proportional representation and apportionment methods are a wonderful example to illustrate the *universitas litterarum*, the unity of arts and sciences.

In retrospect, I find it much easier to conduct an interdisciplinary seminar than to author an interdisciplinary textbook. Nevertheless, I hope that this book may prove a useful reference work for apportionment methods, for scholars of constitutional law and political sciences as well as for other electoral system designers. The many apportionment methods studied span a wide range of alternatives in Germany, the European Union, and elsewhere. The book not only describes the mechanics of each method but also lists the method's properties: biasedness in favor of stronger parties at the expense of weaker parties, preferential treatments of groups of stronger parties at the expense of groups of weaker parties, optimality with respect to goodness-of-fit or stability criteria, reasonable dependence on such variables as house size, vote ratios, size of the party system, and so on. These properties are rigorously proved and, whenever possible, substantiated by appropriate formulae.

Since the text developed from notes that I compiled for lectures and seminars, I am rather confident that it can be utilized for these purposes. The material certainly suffices for a lecture course or a student seminar in a curriculum of mathematics, quantitative economics, computational social choice, or electoral system design in the political sciences. I have used parts of the text with particular success in classes for students who are going to be high-school teachers. The chapters presuppose readers with an appreciation for rigorous derivations and with a readiness to accept arguments from scientific fields other than their own. Most chapters can then be mastered with a minimum knowledge of basic arithmetic. Three chapters involve more technically advanced approaches. Chapters 6 and 7 use some stochastic reasoning and Chap. 14 discrete optimization and computer algorithms.

The subject of the book is restricted to the quantitative and procedural rules that must be employed when a proportional representation system is implemented; as a consequence, the book does *not* explicate the qualitative and normative foundations that would be called for when developing a comprehensive theory of proportional representation. As in all sciences, the classification of quantitative procedures starts with the basic methods that later get modified to allow for more ambitious settings. The basic issue is to calculate seat numbers proportionally to vote counts. This task is resolved by divisor methods or by quota methods. Later, geographical subdivisions of the electoral region come into play, as do guarantees for small units to obtain representation no matter how small they are, as do restrictions for stronger groups to limit their representation lest they unduly dominate their weaker partners. In order to respond to these requirements, the basic methods are modified into variants that may achieve an impressive degree of complexity.

When teaching the topic, I soon became convinced that its intricacies can be appreciated only by contemplating real data, that is, data from actual elections in the real world, rather than imaginary data from contrived elections in the academic ivory tower. My Augsburg students responded enthusiastically and set out to devise an appropriate piece of software, BAZI. BAZI has grown considerably since 2000 and has proved an indispensable tool for carrying out practical calculations and theoretical investigations. I would like to encourage readers of this book to use the program to retrace the examples and to form their own judgment. BAZI is freely available from the website www.uni-augsburg.de/bazi.

Acknowledgments

My introduction to the proportional representation problem was the monograph of *Michel Balinski/Peyton Young* (1982). In their book, the authors recount the apportionment history in the House of Representatives of the USA, and then proceed to establish a Theory of Apportionment. This seminal source was soon complemented by the treatise of *Klaus Kopfermann* (1991) who adds the European dimension to the proportional representation heritage. *Svante Janson's* (2012) typescript proved invaluable for specific mathematical questions. These books provide the foundations on which the results of the present work are based.

Several colleagues and friends read parts or all of the initial drafts of this book and proposed improvements. I have benefited tremendously from the critical comments and helpful suggestions of *Paul Campbell*, *Rudy Fara*, *Martin Fehndrich*, *Dan Felsenthal*, *Svante Janson*, *Jan Lanke*, and *Daniel Lübbert*.

Throughout the project, I had the privilege to rely on the advice and inspiration of my colleagues *Karl Heinz Borgwardt*, *Lothar Heinrich*, and *Antony Unwin* in Augsburg University Institute for Mathematics. A special word of thanks is due to my nonmathematical Augsburg colleagues who helped me in mastering the interdisciplinary aspects of the topic. I wish to thank *Günter Hägele* (Medieval History, University Library), *Thomas Krüger* (Medieval History), *Johannes Masing* (Constitutional Law, now with the University of Freiburg im Breisgau), *Matthias Rossi* (Constitutional Law), and *Rainer-Olaf Schultze* (Political Sciences).

The largest debt of gratitude is due to the current and former members of my workgroup at Augsburg University. Many of them contributed substantially to this work through their research work and PhD theses. Moreover, they helped in organizing lectures and seminars, in sorting the material, in optimizing the terminology, and in polishing the presentation. For their cooperation, I am extremely grateful to *Olga Birkmeier*, *Johanna Fleckenstein*, *Christoph Gietl*, *Max Happacher*, *Thomas Klein*, *Sebastian Maier*, *Kai-Friederike Oelbermann*, *Fabian Reffel*, and *Gerlinde Wolsleben*.

I would like to thank *Andrew Duff*, MEP, for graciously consenting to contribute the foreword to this book. Finally, I wish to acknowledge the support of the Deutsche Forschungsgemeinschaft.

Augsburg, Germany Friedrich Pukelsheim
October 2013

Notations

$\mathbb{N}$	Set of natural numbers $\{0, 1, 2, \ldots\}$ (Sect. 3.1)
$\lfloor t \rfloor$, $\llbracket t \rrbracket$	Floor function (Sect. 3.2); rule of downward rounding (Sect. 3.4)
$\lceil t \rceil$, $\lceil\!\lceil t \rceil\!\rceil$	Ceiling function, rule of upward rounding (Sect. 3.5)
$\langle t \rangle$, $\langle\!\langle t \rangle\!\rangle$	Commercial rounding (Sect. 3.6); rule of standard rounding (Sect. 3.7)
$s(n), n \in \mathbb{N}$	Signpost sequence (always $s(0) = 0$) (Sect. 3.8)
$\llbracket t \rrbracket$, $[t]$	General rounding rule, general rounding function (Sect. 3.9)
$s_r(n) = n - 1 + r$	Stationary signposts ($n \geq 1$) with split parameter $r \in [0; 1]$ (Sect. 3.10)
$\widetilde{s_p}(n)$	Power-mean signposts with power parameter $p \in [-\infty; \infty]$ (Sect. 3.11)
$h \in \mathbb{N}$	House size (Sect. 4.1)
$\ell \in \{2, 3, \ldots\}$	Number of parties entering the apportionment calculations (Sect. 4.1)
$k \in \{2, 3, \ldots\}$	Number of districts (Sect. 15.1)
$v = (v_1, \ldots, v_\ell)$	Vector of vote weights $v_j \in [0; \infty)$ for parties $j \leq \ell$ (Sect. 4.1)
$v_+ = v_1 + \cdots + v_\ell$	Component sum of the vector $v = (v_1, \ldots, v_\ell)$ (Sect. 4.1)
$w_j = v_j / v_+$	Vote share of party j (Sect. 4.1)
$\mathbb{N}^\ell(h)$	Set of seat vectors $x \in \mathbb{N}^\ell$ with component sum $x_+ = h$ (Sect. 4.1)
A	Apportionment rule (Sect. 4.2); apportionment method (Sect. 4.4)
$A(h; v)$	Set of seat vectors for house size h and vote vector v (Sect. 4.2)
$n+ = \{n, n + 1\}$	Upward tie, increment option (Sect. 4.8)
$n- = \{n - 1, n\}$	Downward tie, decrement option (Sect. 4.8)
v_+ / h	Votes-to-seats ratio, also known as Hare-quota (Sect. 5.2)

$w_j\, h$	Ideal share of seats for party $j \leq \ell$ (Sect. 7.1)
$x \preceq y$	Majorization of vectors (Sect. 8.2)
$A(h;v) \preceq B(h;v)$	Majorization of sets of vectors (Sect. 8.3)
$A \prec B$	Majorization of apportionment methods (Sect. 8.4)
$a = (a_1, \ldots, a_\ell)$	Minimum requirements (Sect. 12.3)
$b = (b_1, \ldots, b_\ell)$	Maximum cappings (Sect. 12.3)
$:=$	Definition
$\square$	End-of-proof
-ward	Suffix of adjectives: the downward rounding, etc.
-wards	Suffix of adverbs: to round downwards, etc.
$\bullet$	Multipurpose eye-catcher in tables

Outline of the Book

Chapters 1 and 2: Apportionment Methods in Practice

The two initial chapters present an abundance of apportionment methods used in practice. Chapter 1 reviews the European Parliament elections of 2014, providing a rich source of examples. Chapter 2 deals with the German Bundestag election of 2009; emphasis is on the interplay between procedural steps and constitutional requirements. The chapters introduce concepts of proportional representation systems that prove crucial beyond their specific use in European or German elections. Concepts and terminology in the initial chapters set the scene for the methodology that is developed in the sequels.

Chapters 3–5: Divisor Methods and Quota Methods

A rigorous approach to apportionment needs to appeal to rounding functions and rounding rules. They are introduced in Chap. 3. Chapters 4 and 5 discuss the two dominant classes of apportionment methods: divisor methods and quota methods. Usually, the input vote counts are much larger than the desired output seat numbers. Therefore, vote counts are scaled down to interim quotients of a fitting magnitude. Then, the interim quotients are rounded to integers. Divisor methods use a flexible divisor for the first step and a specific rounding rule for the second. Quota methods employ a formulaic divisor—called quota—for the first step and a flexible rounding rule for the second.

Chapters 6–8: Deviations from Proportionality

Many apportionment methods deviate from perfect proportionality in a systematic fashion. Chapters 6 and 7 investigate seat biases. A seat bias is the average of the deviations between actual seat numbers and the ideal share of seats, assuming that all vote shares are equally likely or that they follow an absolutely continuous distribution. Chapter 8 compares two apportionment methods by means of the majorization ordering. The majorization relation allows comparison of two apportionment methods. The relation indicates whether one method is more beneficial to groups of stronger parties—and hence more disadvantageous to the complementary group of weaker parties—than the other method.

Chapters 9–11: Coherence, Optimality, and Vote Ranges

Chapter 9 explores the idea that a fair division should be such that every part of it is fair, too. This requirement is captured by the notion of coherence. Divisor methods are coherent; quota methods are not. Chapter 10 evaluates goodness-of-fit criteria to assess the deviations of actual seat numbers from ideal shares of seats. Particular criteria lead to particular apportionment methods. Chapter 11 reverses the role of input and output. Given a seat number, the range of vote shares leading to the given seat number is determined. The results elucidate situations when a straight majority of votes fails to lead to a straight majority of seats. As a corrective, many electoral laws include an extra majority preservation clause. Three majority clauses are discussed, and their practical usage is illustrated by example.

Chapters 12 and 13: Practical Implementations

Many electoral systems impose restrictions on seat numbers. Chapter 12 shows how to handle minimum requirements as well as maximum cappings. The practical relevance of restrictions is shown by examples, such as the allocation of the seats of the European Parliament between the Member States of the Union. Chapter 13 discusses the 2013 amendment of the German Federal Election Law. The system realizes practical equality of the success values of all voters in the whole country. Mild deviations from proportionality, due to direct-seat restrictions, are incurred when assigning the seats of a party to its lists of nominees.

Chapters 14 and 15: Double Proportionality

Double proportionality aims at a fair representation of the geographical division of the electorate as well as of the political division of the voters. The methods achieve this two-way fairness by apportioning seats to districts proportionally to population figures and to parties proportionally to vote counts. The core is the sub-apportionment of seats by district and party in such a way that, for every district, the seats sum to the given district magnitude and, for every party, the seats sum to their overall proportionate due. To this end, two sets of electoral keys are required: district divisors and party divisors. While it is laborious to determine the electoral keys, their publication makes it rather easy to verify the double-proportional seat apportionment. Chapter 14 explains double-proportional divisor methods by example; Chap. 15 adjoins the necessary theory.

Chapter 16: Biographical Digest

Homage is paid to selected individuals who contributed to the genesis of apportionment methods for use in proportional representation systems.

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