Gustav Elfving: An Appreciation

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Erik Gustav Elfving (1908–1984) played an important role in the development of statistics and probability in Finland and elsewhere. His life and achievements are celebrated in this article.

Part I: Biographical notes

Erik Gustav Elfving was born on June 25, 1908 in Helsinki, Finland to parents Fredrik and Thyra Elfving, née Ingman. He was the youngest of four children; there were two daughters and two sons. It appears that the first name Erik was not popular with the recipient as he always called himself Gustav Elfving, except in publication[ž] [4-1954b] where the full name is listed. Among family and friends, he was affectionately referred to as Gusse.

His father was for many years a Professor of Botany at the University of Helsinki with a reputation for being a demanding teacher. Indeed, it is told that students who had to pass the feared oral examination by Fredrik Elfving remembered with awe the scrutiny with which their knowledge and creativity

AMS 1991 *subject classifications.* 01A70, 60–03, 62–03
Key words. Complex variables, Design of experiments; Game theory; Historical figures in statistics; Least squares; Markov chains and processes; Probability; Sequential analysis; Statistics in Finland.

ž Elfving's publications are grouped by topics in Sections 1–12 of Part II below. We quote an item in the format [n-yyyy], the prefix pointing to Section n if different from the current section. We also give the location of reviews in *Mathematical Reviews* (MR) and *Zentralblatt für Mathematik* (Zbl) when applicable. In retrospect it appears to us that the reviews, in their factual, noncommittal style, have little predictive value for the worth of Elfving's writings.

was tested, even decades after the event took place! The father was also a very energetic person. While in his twenties, he had raised money in Finland for a London memorial to Charles Darwin; see Klinge, Knapas, Leikola and Strömberg (1989). Later, he championed the creation of a new building for the Institute of Botanics in the Botanical Garden, in which he both lived and worked. Gustav spent his childhood in the Professor's residence in the Botanical Garden, a good beginning for any academic!

The pursuit of an academic career must thus have been a natural choice for the gifted young man. In 1926 he enrolled at the University of Helsinki, planning to major in astronomy. While attending mathematics courses for astronomy students, his interest for mathematics was aroused. Consequently, he switched to mathematics, graduating in 1930 in mathematics, with astronomy and physics as minor subjects. His 1934 PhD in mathematics was awarded for research in complex analysis, specifically in what is now called the Nevanlinna value distribution theory. His adviser, we are not surprised to learn, was one of the two Nevanlinna brothers, Rolf. Many years later, Elfving was to deliver memorial lectures [12-1977, 12-1980a] for both Rolf Nevanlinna (1895–1980) and Frithiof Nevanlinna (1894–1977).

Other influential teachers were Ernst Lindelöf (1870–1946) and Jarl Waldemar Lindeberg (1876–1932), who introduced Elfving to probability theory. Lindeberg's results on the Central Limit Theorem still belong in any study program on probability. Elfving's own story of his beginning research in probability and statistics involves a cartographic expedition of the Danish Geodetic Institute, with which he traveled in 1935 to Western Greenland. Because of incessant rain, the party was forced to spend three entire days in their tents. What better circumstances could there be for thinking about least squares problems!

Elfving's first permanent academic post was as a lecturer at Åbo Akademi in Åbo (Turku), Finland, from 1935 to 1938. In 1936 he married Irene (Ira), née Aminoff. They had three children, all boys (Johan, Jörn and Tord, born in 1938, 1941 and 1946, respectively).

Between 1938 and 1948 he was a lecturer at the Helsinki University of Technology, and from 1948 to 1975 he was Professor of Mathematics at the University of Helsinki, succeeding Lars Ahlfors who had taken up a position at Harvard University.

During World War II he served in the Finnish coast artillery as a geodesist, and when possible also taught mathematics. This period also marks the beginning of his life-long interest in the writing of articles directed to a wide non-specialist audience [10-1941, 10-1942a, 10-1945b].

On several occasions he took leave from Helsinki and went to

universities abroad. During the academic year 1946–1947 he was locum tenens professor at Stockholm University, substituting for Harald Cramér, who had been invited to lecture at Princeton University in the fall of 1946 (when Princeton celebrated its Bicentennial Year) and at Yale University in the spring of 1947. Elfving spent several extended periods in the United States. At the invitation of William Feller, he spent the period 1949–1951 at Cornell University, where he shared an office with Monroe Donsker. He spent the spring term 1955 at Columbia University, at the invitation of T.W. Anderson, Herbert Robbins and Herbert Solomon. The fall 1960 and spring 1966 semesters he spent at Stanford University, at the invitation of Herbert Solomon.

During the stay in 1955 he was an invited speaker at the Third Berkeley Symposium, and also visited, among others, the University of Chicago. Indeed, later on both Leonard J. Savage and William H. Kruskal of that university were to visit the Elfvings in Finland, along with many other prominent statisticians and probabilists. Quite a few visitors were taken to Sundholm, the summer residence of the Elfvings, located near Nystad (Uusikaupunki) on the western coastline some 70 kilometres northwest of the city of Åbo. The old parts of Sundholm, which is a large mansion belonging to his wife's family (the Aminoffs), date back to the year 1487, and it is said that Gustav Elfving occasionally joked with his American visitors remarking that Sundholm was built five years before Christopher Columbus arrived in the New World!

During all of these stays in the United States, Elfving was a visiting professor with teaching duties besides carrying out research. Among his papers we find the final examination in Mathematical Statistics 244 on 24 May 1955, obviously at Columbia University. There were five students, and the final grades were two A's, two B+'s and a B. It appears that he also set questions on a qualifying exam on 29 April 1955, but no indication appears of which questions were his.

In the 1950's and the 1960's Elfving received several offers for permanent positions from universities in the United States. Although he must have been tempted to move from the relative scientific isolation in Finland to the stimulating environment at major departments in the United States, his characteristic sense of duty and loyalty towards his country and his friends, and his concerns about the family, made him decline these offers. In one of the letters regarding an offer he had received in the 1950's one reads:

> "... there are so many things that keep me here [in Finland], in the long run: ties of friendship and kinship, loyalty towards a small and poor

country, the psychological problems that would arise in transplanting the children, and ourselves, in a new ground."

Indeed, for family reasons and probably also feeling that he should not be away from his Alma Mater for too long a period in a stretch, he visited both Columbia and Stanford for only one term at a time. At Cornell, he was accompanied by his wife and children.

The period from the 1960's until retirement was a period filled with many kinds of activities. Besides the everyday teaching, seminars and guiding of students as well as research, Elfving's expertise was called upon in other academic matters at an increasing rate, both abroad as well as in Finland. He was opponent at numerous doctoral defences (on topics quite distant from his own research), and evaluated on several occasions candidates for professorial chairs, in particular in the Nordic countries. Furthermore, he sat on the editorial board of *The Annals of Mathematical Statistics* 1964–1967 and *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete* 1962–1975, and was the regional editor of *Mathematica Scandinavica* 1953–1972. He also served the Institute of Mathematical Statistics and the International Statistical Institute in various capacities in the 1960's and 1970's.

Although Elfving was a man who would never in discussion have brought forth his own achievements, his scientific contributions to the fields of statistics and probability, and his other services to the profession, did not go without recognition. In addition to being elected to various Finnish scientific societies, he was elected Fellow of the Institute of Mathematical Statistics in 1955, and became an elected member of the International Statistical Institute in 1963. In 1974, he was made an Honorary Fellow of the Royal Statistical Society and a Foreign Member of the Royal Academy of Sciences (Sweden).

When he retired in 1975, he was 67 years old. Retirement meant turning to other things, not cessation of activity. His book *The History of Mathematics in Finland 1828–1918* [12-1981a] clearly involved a great deal of patient research and reads beautifully today. Gustav Elfving died at his home in Helsinki on March 25, 1984, outliving his beloved wife Ira by five years.

In a three page note [10-1970] which appears in *Scientists at Work*, a Festschrift in honor of Herman Wold in 1970, Elfving gives us a glimpse into the way he felt about research:

> "I have practically never participated in real team work. At home, I make suggestions to pupils, listen to them, and read their writings. On research projects abroad, the members of the group have simply picked one subject each. A modest amount of discussion has gone on but, on

> the whole, I have worked alone, sitting at my desk or walking up and down the campus. I have rarely done research work at home, except at my country place [Sundholm]."

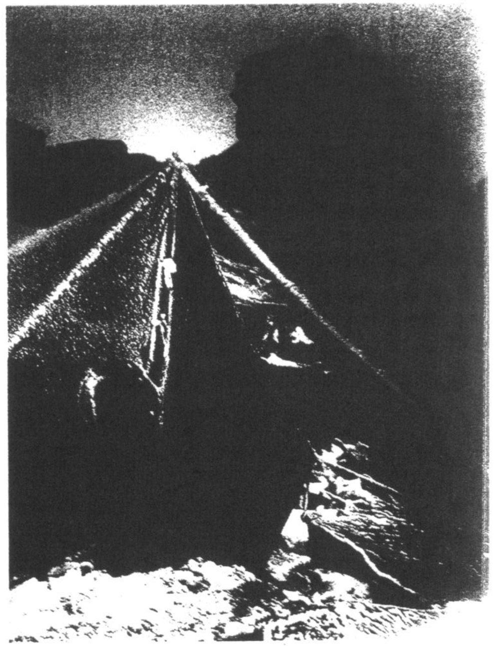
The accuracy of the first two sentences of this quote is confirmed by Johan Fellman in recalling his days as Elfving's PhD student.

In the same note Elfving, who strongly felt that one should really have substantial results of hopefully lasting value before publishing, expresses alarm about the proliferation of scientific writing:

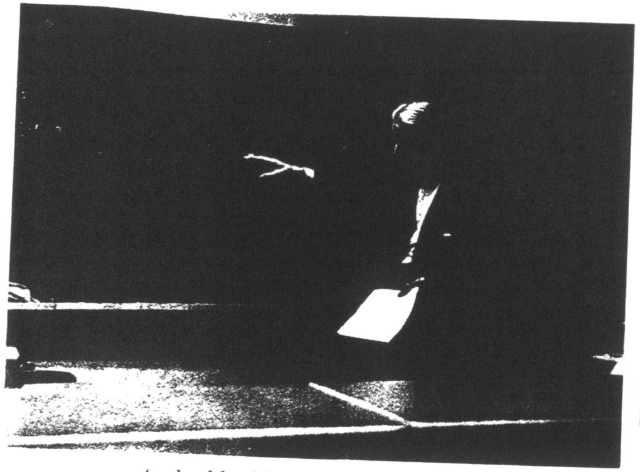
> "I wish that the [unpublished technical] reports could be typed on paper that would automatically decay after five years."

In view of the concerns Elfving repeatedly expressed in private correspondence, he would probably extend the above wish to cover also a large portion of the current journal articles, appearing as offsprings of the present-day rat race, with its quest for least publishable units.

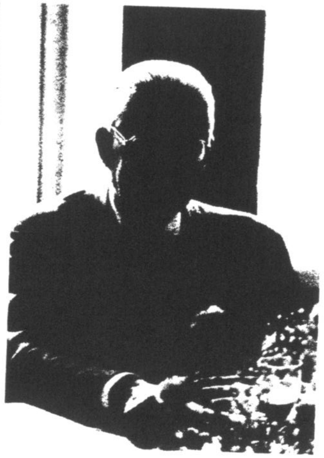
Gustav Elfving was, foremost, a man of learning. He perceived promulgation of ideas as a more important scientific duty than interaction with people. Casual acquaintances observed his courteous reticence, while those who came closer met an unusually considerate, courteous and warm human being, an open-minded man who enjoyed conversation and good company, and contributed his wit, humor and charm. He had exceptionally broad interests in philosophy, history, literature, languages, and society at large. He wrote poems and enjoyed philosophical discussions, for example, with his close friend Georg Henrik von Wright, the Finnish philosopher who in 1948 was appointed the successor of Ludwig Wittgenstein at Cambridge University. Nearly every summer from the beginning of the 1950's onwards he would go sailing for a week or two, writing poems and enjoying the unique Finnish archipelago in the



Camping in Greenland



At the blackboard



Birthday party

company of von Wright and Eric Bargum, a life-long friend, mathematician and the captain of the boat. Besides Swedish (his native language) and Finnish, Gustav Elfving had an excellent command of German, Latin, French, and English.

The present paper is partly based on the biographical notes and chronological list of Elfving's scientific writings in Mäkeläinen (1984, 1985, 1997), and the material in Fellman (1991, 1999) and Pukelsheim (1993, page 430). Some complementary aspects of Elfving's life and scientific work, including an updated chronological list of publications, are given in Nordström (1999).

Part II: Bibliography with annotations

1. Complex Variables

[1934] Über eine Klasse von Riemannschen Flächen und ihre Uniformisierung. *Acta Societatis Scientiarum Fennicæ, Nova Series A* **2**(3), 60 pages. Zbl 010.36301

[1935a] Über Riemannsche Flächen und Annäherung von meromorphen Funktionen. In *Åttonde skandinaviska matematikerkongressen i Stockholm 1934*. Håkan Ohlssons Boktryckeri, Lund, 96–105. Zbl 012.08002

[1935b] Zur Flächenstruktur und Wertverteilung. Ein Beispiel. *Acta Academiae Aboensis, Mathematica et Physica* **8**(10), 13 pages. Zbl 011.31302

Elfving wrote his Dissertation [1934] under the supervision of Rolf Nevanlinna, on the inverse problem of the Nevanlinna theory. The inverse problem is to construct a meromorphic function with prescribed deficiency and ramification indices. For a review and further advances see Drasin (1977), in which Elfving's work is praised on page 85.

In the Introduction, Elfving expresses his gratitude to Rolf Nevanlinna for setting him to work on this topic and for valuable assistance, as well as to his "highly esteemed teacher" Ernst Lindelöf for the "untiring interest" with which he has followed Elfving's studies. Papers [1935a,b] further illustrate the results from the Dissertation, by means of some specifically selected examples.

2. Markov Chains and Point Processes

[1937] Zur Theorie der Markoffschen Ketten. *Acta Societatis Scientiarum Fennicæ, Nova Series A* **2**(8), 17 pages. Zbl 017.31603

[1938] Über die Interpolation von Markoffschen Ketten. *Societas Scientiarum Fennica, Commentationes Physico–Mathematicae* **10**(3), 8 pages. Zbl 018.26403

[1946a] Contributions to the theory of integer-valued Markoff processes. *Skandinavisk Aktuarietidskrift* **29** 175–205. MR 8.391c

[1946b] On compound binomial processes. In *Försäkringsmatematiska studier tillägnade Filip Lundberg*. Almqvist & Wiksell, Uppsala, 48–78. MR 8.391b

[1947] On a class of elementary Markoff processes. In *Den 10. skandinaviske matematiker kongres i København 1946*. Jul. Gjellerups Forlag, Copenhagen, 149–159. MR 8.472f, Zbl 030.40202

[1967] A persistency problem connected with a point process. *Journal of Applied Probability* **4** 77–89. MR 37.3695, Zbl 183.20705

In the first two papers [1937, 1938] Elfving discusses Markov chain problems, a topic that is disjoint from his Dissertation theme. Elfving starts out from work of Kolmogorov and others that had appeared only a year earlier. He points out that most papers on Markov chains aim at determining the stationary distribution of states. In contrast, he proposes two questions of a different sort. First, is it possible to embed a discrete time Markov chain in a continuous time Markov chain? Elfving shows that the problem may have no solution, a unique solution, or finitely many solutions.

The second question takes as a starting point that, given the state distribution $Q^{(0)}$ at time t_0, either the discrete time transition probability matrix P or the continuous time infinitesimal generator A permit to determine the state distributions at any subsequent point in time $t > t_0$. But what about previous times $t < t_0$? This means asking whether the process possibly extends into an infinite past (yes, when $Q^{(0)}$ is the stationary distribution), or whether it has started at some definite past time $\tau > -\infty$ (yes, otherwise).

For time-homogeneous Markov chains the answers are given in [1937]. The paper is written in German, but was translated into English and was issued in the Mimeo. Series, No. 103 (May 1954), Department of Experimental Statistics, North Carolina State College. We have no evidence of who took the initiative, or who claimed interest to work with an English version. Paper [1938] relieves the assumption of time homogeneity.

The second set of papers [1946a,b, 1947] appeared about ten years later. Elfving now views Markov processes as a particular class of point processes. Following earlier work of Ove Lundberg, the first of the three papers delineates the general theory to obtain a compound representation of the process in question, and to calculate the asymptotic distributions of the "saltus points". Paper [1946b] illustrates the result for the special classes of compound Binomial or Poisson processes. The conference paper [1947] exemplifies ideas in a transparent discrete-time setting using the Pólya–Eggenberger urn scheme.

Twenty years later, Elfving returns to stochastic processes and solves a stopping problem [1967]:

> "Imagine a man owning a commodity, e.g., a house, which is for sale. Offers at varying amounts are coming in every now and then. The longer he postpones selling the more he loses because of deterioration, interest losses, or the like. At each offer he must decide whether to accept it or wait for a better one. (A more picturesque example would be that of a girl scrutinizing successive suitors.)"

An integral equation and a differential equation for the critical curve are derived. Under a certain convergence assumption, the existence and uniqueness of an optimal decision rule is established, and several illustrations are given. In the case of divergence, various possibilities are discussed.

3. Sampling Distributions from Normal Populations

[1947a] The asymptotical distribution of range in samples from a normal population. *Biometrika* **34** 111–119.
MR 8.395b, Zbl 030.16802

[1947b] A simple method of deducing certain distributions connected with multivariate sampling. *Skandinavisk Aktuarietidskrift* **30** 56–74.
MR 9.48h, Zbl 030.40501

[1955] An expansion principle for distribution functions with application to Student's statistic. *Annales Academiæ Scientiarum Fennicæ, Series A I, Mathematica–Physica* **204**, 8 pages.
MR 17.981d, Zbl 066.11903

Both 1947 papers determine sampling distributions in normal populations, but with quite a different outlook. It almost looks like a paradigm shift in action. In [1947a], deducing the distribution means, in the narrow interpretation that was then predominant, finding the probability density function and the cumulative distribution function. In contrast, the goal in [1947b] is more liberal:

> "Our results will, as a rule, be presented in terms of random variables rather than distribution functions ... Such a representation seems in many cases more useful than an explicit distribution law: on the one hand, it allows a direct and simple calculation of the moments; on the other, it may, owing to the powerful modern devices for mechanical integration, provide the most efficient tool for numerical evaluations."

The development in [1947a] could be outlined as follows. For an i.i.d. sequence $X_1, X_2, \ldots$ of standard normal random variables, let A_n and B_n denote the smallest and the largest value among $X_1, \ldots, X_n$. The appropriately standardized sequence (A_n, B_n) converges in distribution to (Y, Z), where Y and Z are independent random variables with an extreme value type-1-(or Gumbel-)distribution. Thus the appropriately standardized sample range $W_n = B_n - A_n$ converges in distribution to $Y - Z$, see, for example, Witting and Müller-Funk (1995, page 596).

Elfving [1947a], however, uses the transformation $U_i = \Phi(X_i)$ to study $\Phi(A_n)$ and $\Phi(B_n)$ instead. In this, he follows "the example of the [previous] authors" – without mentioning that this probability transform yields uniform variates, so that the limiting variates Y and Z now each have an extreme value type-3-(or Weibull-)distribution. Referring to special functions of classical calculus, Elfving determines and tabulates the probability density function and the cumulative distribution function of the limiting distribution of $\Phi(W_n)$. The result is not mentioned in Johnson, Kotz and Balakrishnan (1995). Those authors quote (*ibid.*, page 87) Gumbel (1949) who, however, explicitly and at length refers back to Elfving [1947a].

Elfving's approach in [1947b], to use alternative matrix transformations in order to express multivariate distributions in terms of familiar univariate distributions, is now a standard tool in multivariate statistical analysis, see Anderson (1984, page 251).

Elfving [1955] derives an approximation to the Student t_n-distribution, by calculating terms to correct the limiting normal distribution for finite degrees of freedom n. Here he builds on his experience from his PhD years; a statement like "converging in the whole complex plane since [it] is an entire function" flows smoothly from his pen. The result is reproduced in Johnson, Kotz and Balakrishnan (1995, page 377). We note, in passing, that a 1952 paper by Sitgreaves, a coauthor of [5-1959] is referenced here.

4. Design of Experiments

[1952] Optimum allocation in linear regression theory. *Annals of Mathematical Statistics* **23** 255–262.
MR 13.963h, Zbl 047.13403

[1954a] Convex sets in statistics. In *Tolfte skandinaviska matematikerkongressen i Lund 1953*. Håkan Ohlssons Boktryckeri, Lund, 34–39.
MR 16.499c, Zbl 056.37201

[1954b] Geometric allocation theory. *Skandinavisk Aktuarietidskrift* **37** 170–190.
MR 17.640d, Zbl 068.13303

See also abstract "A unified approach to the allocation problem in sampling theory" prepared for a short lecture; in *Proceedings of the International Congress of Mathematicians, Amsterdam 1954* (Eds. J.G.H. Gerretsen, J. de Groot). Wiskundig Genootschap, Amsterdam, **2** 288–289.

[1956] Über optimale Allokation. In *Bericht über die Tagung Wahrscheinlichkeitsrechnung und mathematische Statistik, Berlin 1954* (Ed. B.W. Gnedenko). VEB Deutscher Verlag der Wissenschaften, Berlin, 89–95.
MR 18.425h, Zbl 070.14407

[1958] Minimax character of balanced experimental designs. In *Trettonde skandinaviska matematikerkongressen i Helsingfors 1957*. Mercators Tryckeri, Helsinki, 69–76.
MR 21.4514, Zbl 088.11802

[1959] Design of linear experiments. In *Probability and Statistics – The Harald Cramér Volume* (Ed. U. Grenander). Almqvist & Wiksell, Stockholm, and Wiley, New York, 58–74.
MR 22.1970, Zbl 119.35202

[1961] Designs for quadratic regression analysis (with discussion). In *EU CE PA, Fourth Symposium: Statistical Methods in the Pulp and Paper Industry, Helsinki 1960*. Finnish Paper Engineers' Association, Helsinki, 149–154.

[1963] Geometric and game-theoretic approaches to optimum allocation. Chapter 14 in *Mathematical Optimization Techniques* (Ed. R. Bellman). University of California Press, Berkeley and Los Angeles, 291–299.
Zbl 117.14404

The work on the design of experiments had the greatest impact out of Elfving's scientific œuvre. What made him shift his field of interest to optimal design of experiments? In [10-1970] he reports, evidently referring to an incident during his 1949–1951 visit to Cornell University:

> "... a seminar talk at the School of Agriculture at Cornell University brought up a design problem which led me to a series of studies on optimal allocation."

The PhD work [1-1934] on complex functions already demonstrates his interest in the interplay between analytical methods and geometric interpretation. Also he handles the transition probability matrix of a Markov chain [2-1937], and the matrices in a linear model [3-1947b] in a masterful, algebraic way. Both geometric insight and algebraic skills come to bear in Elfving's

work on optimal design of experiments.

Paper [1952] on "Optimum allocation in linear regression theory" marks the beginning of the optimality theory of experimental design. Elfving starts out from a linear model of the specific form

$$y_i = x_{i1}\beta_1 + x_{i2}\beta_2 + \eta_i,$$

where y_i denotes the observed response, $x_i = (x_{i1}, x_{i2})'$ the regression vector, $\beta = (\beta_1, \beta_2)'$ the unknown parameter vector for the expected response, and η_i the error term. For $i = 1, \ldots, r$ it is assumed that the error terms are stochastically independent, and all have mean zero and the same standard deviation. Furthermore the experimenter may repeat each experimental run as many times as he pleases, or not at all.

If the experimenter has decided upon a certain total n of actual observations, he is faced with the problem of which of the potential runs to perform, and in what number.

Let the required number of observations on the regression vector x_i be n_i. Then the proportions $p_i = n_i/n$ fulfill the conditions

$$p_i \geq 0 \quad \text{and} \quad \sum_{i=1}^{n} p_i = 1.$$

Elfving makes the important point that, for large sample sizes n, the proportions p_i may be varied practically continuously, rather than being restricted to be multiples of $1/n$. With this transition from a discrete to a continuous optimization problem, Elfving lays the foundation for a powerful optimality theory for the design of experiments.

Whether the optimality problem can be solved is, of course, contingent on the choice of the optimality criterion. In Section 2 of the [1952] paper, Elfving solves the problem for the estimation of a single linear combination of the parameters, $a'\beta = a_1\beta_1 + a_2\beta_2$. In this case of a scalar parametric function, the one and only natural optimality criterion calls for determining the proportions p_i so as to yield the smallest variance for the least-squares estimator $a'\hat{\beta} = a_1\hat{\beta}_1 + a_2\hat{\beta}_2$.

Elfving's solution is geometric in nature, involving the study of the convex polygon Π that is spanned by the vectors $\pm x_1, \ldots, \pm x_r$. If X_1 and X_2 denote the vertices of Π between which the ray A that is defined by the coefficient vector a penetrates the boundary, then X_1 and X_2 are the regression vectors which specify where to take the observations, and the proportions p_1 and p_2 must be in the same ratio in which the point of intersection A^*

subdivides the line connecting X_1 and X_2. See Figure 1.

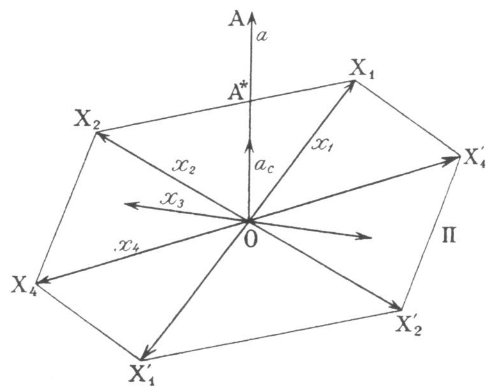


Fig. 1

Studden (1971) appears to have been the first to refer to this result as "Elfving's Theorem". The polygon Π and its generalizations are nowadays called the "Elfving set". Elfving's geometric approach is a powerful tool that carries over to more demanding objective functions when some, or all parameters are of interest, rather than a single scalar combination. An important contribution in this direction is the thesis of Johan Fellman (1974). For more recent references, see Dette, Heiligers and Studden (1995).

In fact, in Section 3 of [1952], Elfving treats the estimation of both parameters in his specific model. He chooses what today is called the average-variance criterion, that is minimizing the trace of the covariance matrix of the least-squares estimator $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2)'$. Elfving's geometric approach leads to a condition that is necessary only, namely that all optimum regression vectors *xi* lie on an ellipse centered at the origin, while none of the other regression vectors lie outside of it. In a sense, the ellipse has a minimum "size" – as measured by a dual optimality criterion – among all those ellipses that contain the Elfving set Π. In pure mathematics, such an ellipsoid is called the Loewner ellipsoid, see Pukelsheim (1993, page 417) and the references given there. In statistics, the duality between the Elfving set and its covering ellipsoid provides a geometric interpretation of the General Equivalence Theorem.

Elfving must have felt the importance of his results, because he reports on them at congresses in Lund 1953, Berlin 1954, and Amsterdam 1954 [1954a,b, 1956]. Moreover, paper [1954a] contains the best known, general upper bound for the number of observations when estimating s out of k parameters, namely

$$k + (k-1) + \cdots + (k-s+1).$$

This result was found independently by Chernoff (1953, page 595). Elfving's arguments are geometric in nature, Chernoff's are algebraic. While Elfving and Chernoff use the then predominant average-variance criterion, the upper bound is actually valid in a much wider context of virtually all reasonable optimality criteria, see Fellman (1974, page 62).

Paper [1954b], which was presented at the 1954 Amsterdam ICM, is a careful exposition of Elfving's approach. It emphasizes that the optimality problem is best formulated in terms of moment matrices M that vary over a convex set $\mathcal{M}$.

Elfving [1956] points out that his results might be of interest much beyond the field of the design of experiments. He mentions applications in geodesy, as well as for sampling from finite populations. He indicates that the average-variance criterion is just one possible choice of an optimality criterion, another one might be to minimize the largest among the variances of the components of the least-squares estimator. Or when part of the observations are already on record, one may ask how to augment these in an optimum way.

Elfving himself indicates the wide applicability of his approach in [1958], where he discusses optimality of balanced incomplete block designs in the one-way classification model with additive treatment effects and block effects,

$$y_{ijk} = \tau_i + \beta_j + \eta_{ijk}.$$

In fact, the arguments are very much in line with those Kiefer (1975) used to motivate the concept of universal optimality of block designs. It seems, however, that this paper [1958] evaded Kiefer's (1975) attention. A more classical application, to quadratic regression, is discussed by Elfving in [1961].

Elfving's contribution to the Harald Cramér Volume [1959] is a definitive summary of his work on the design problem. Notation and assumptions are carefully selected. An experiment is referred to in two parts, the "spectrum" $(a_1, \ldots, a_r)$ of those regression vectors a_i that are going to be realized, and the "allocation" $(p_1, \ldots, p_r)$ of proportions $p_i = n_i/n$ that determine the number of independent replications. The set of all regression vectors is denoted by $\mathcal{R}$, and the set of corresponding moment matrices by $\mathcal{M}$. In either case, Elfving soon makes a transition to the respective convex hulls $\overline{\mathcal{R}}$ and $\overline{\mathcal{M}}$.

After reviewing his results on a scalar parameter function, Elfving [1959] turns to the case where more than one parameter has to be estimated. He considers quadratic loss, and a minimax criterion in the sense that the optimal design minimizes the maximum component variance of the least-squares estimator. When the maximum is extended over all components, Elfving speaks of "total minimaxity". In the presence of nuisance parameters, when only a subset of parameters is of interest, he uses the term "partial minimaxity". A specific application is to the one-way classification model with additive treatment effects and block effects. For an example in the exact optimality design theory, see Krafft and Schaefer (1995).

Following the lines of general decision theory, Elfving [1959] also introduces the notion of "total admissibility" and "partial admissibility". He finds that total admissibility is a property of the spectrum alone, and closely related to optimality relative to some quadratic loss function. These points are later elaborated upon by Karlin and Studden (1966, page 808). Elfving emphasizes that these conclusions break down for partial admissibility, leaving only the fairly elementary result that the regression vectors in an optimal spectrum must be extreme points of an appropriate polyhedron.

Elfving concludes his [1959] paper by pointing out that the main problem left unsolved is to find numerical procedures to replace the geometrical methods that are available when the parameter vector has two or three components. In a "Note added in proof" he mentions that he meanwhile had access to a forthcoming article by J. Kiefer and J. Wolfowitz disclosing direct connections between optimum allocation and game theory, and opening new possibilities for treating the cases with more than three parameters.

Paper [1963] appears to be a direct sequel in this program. It is Elfving's attempt to reconcile his geometric approach with the game-theoretic method of Kiefer and Wolfowitz. Maybe the results were not sufficiently concrete to really please Elfving, since this appears to be his last publication on the topic of optimum design of experiments. The point is that Nature's strategies do not form a convex set, whence a transition to randomized strategies is called for in order to apply the fundamental theorem of game theory. As Elfving comments:

> "The carrying out of this program may be no less complex than the original problem."

It is, however, precisely along this route that Heiligers (1996, page 365) succeeded in obtaining designs that optimize the smallest-eigenvalue criterion.

5. Psychometrics

[1956] Selection of nonrepeatable observations for estimation. In *Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability, Berkeley 1954 and 1955* (Ed. J. Neyman). University of California Press, Berkeley, **1** 69–75. Also Chapter 6 "Item selection and choice of nonrepeatable observations for estimation" in *Studies in Item Analysis and Prediction* (Ed. H. Solomon). Stanford University Press 1961, 88–95.
MR 18.946d, 23.A737, Zbl 072.36303, 131.36103

[1957] A selection problem in experimental design. *Societas Scientiarum Fennica. Commentationes Physico-Mathematicae* **20**(2), 10 pages. Also Chapter 5 "The item-selection problem and experimental design" in *Studies in Item Analysis and Prediction* (Ed. H. Solomon). Stanford University Press 1961, 81–87. MR 21.405, 23.A739, Zbl 078.33601, 131.36201

[1959] (With R. Sitgreaves and H. Solomon) Item selection procedures for item variables with a known factor structure. *Psychometrika* **24** 189–205. Also Chapter 4 in *Studies in Item Analysis and Prediction* (Ed. H. Solomon). Stanford University Press 1961, 64–80.
MR 21.6054, 22.12638, Zbl 118.35202, 125.38301

[1961a] Contributions to a technique for item selection. Chapter 7 in *Studies in Item Analysis and Prediction* (Ed. H. Solomon). Stanford University Press 1961, 96–108.
MR 23.A738, Zbl 131.36104

[1961b] An expansion principle for distribution functions with applications to Student's statistic and the one-dimensional classification statistic. Chapter 17 in *Studies in Item Analysis and Prediction* (Ed. H. Solomon). Stanford University Press 1961, 276–284. MR 23.A4209, Zbl 149.15805

The papers in this section were published between 1956 and 1961. In fact some of them were published twice in that period! When Elfving visited Columbia University in New York City in Spring 1955, he joined a research team formed to carry out financed research for the United States Air Force School of Aviation Medicine in Texas, under a contract that began in October 1954. The research group included Theodore W. Anderson, Raj R. Bahadur, Allan Birnbaum, Albert Bowker, Gustav Elfving, M. Vernon Johns, Howard Levene, Edward Paulson, Howard Raiffa, Herbert Robbins, Rosedith Sitgreaves, Herbert Solomon and Daniel Teichrow. Paul Lazarsfeld and Irving Lorge also contributed to the effort. The five papers which were written by Elfving (one of these with two coauthors) all appeared as chapters in the book *Studies in Item Analysis and Prediction*, edited by Herbert Solomon and published by Stanford University Press in 1961.

Chapter 4 [1959] assumes that item variables $u_1, \ldots, u_N$, test results in psychometric studies, are directly observable and that a criterion variable, such as a score measuring the capability of a patient, can be determined from the item variables essentially by an estimate which is a linear function of $u_1, \ldots, u_N$. The experimenter can select n of the N potential experiments. Which should he choose? The theory of Chapters 5–7 is brought to bear on this problem, and two small examples, one real, one constructed, are discussed.

Chapter 5 [1957] restates the problem in a form akin to the optimum allocation problem. Interest concentrates on the generalized information matrix $M = \sum_{i=1}^{N} p_i u_i u_i'$, where

$$0 \le p_i \le 1 \quad \text{and} \quad \sum_{i=1}^{N} p_i = n.$$

The main theorem then states that $c'M^{-1}c$ is minimized by choosing boundary points among the vectors $u_1, \ldots, u_N$, some fully and some (at most k of them) fractionally.

Chapter 6 [1956] extends the above work to the case where the item points $u_1, \ldots, u_N$, represent an "idealized" distribution function $f(u)$ rather than discrete points. The extended form of the previous theorem makes it possible to find conditions on the design moments, the elements of M, for an optimum design. Chapter 7 [1961a] is a mathematically dense continuation of Chapter 6, giving a "tentative technique" for improving non-optimal designs.

Chapter 17 [1961b] goes off on a different topic. Elfving develops an asymptotic expansion for the cumulative distribution function of T.W. Anderson's classification statistic along the lines of his [3-1955] paper which, in fact, is reproduced as Sections 1 and 2.

6. Biometrics, Hydrography and Quality Control

[1930] Zur Reduktion von Echolotungen. *Havsforskningsinstitutets Skrift* **69**, 11 pages.

[1944] (With I. Hustich) Die Radialzuwachsvariationen der Waldgrenzkiefer. *Societas Scientiarum Fennica, Commentationes Biologicae* **9**(8), 18 pages.

[1950] (With J.H. Whitlock) A simple trend test with application to Erythrocyte size data. *Biometrics* **6** 282–288.

[1957] Statistical analysis of incomplete material. Appendix pp. 9–11 to Lars von Haartman's paper "Population changes in the Tufted Duck, Aythya fuligula (L.)." *Societas Scientiarum Fennica, Commentationes Biologicae* **16**(5), 11 pages.

[1962a] The AOQL of multi-level continuous sampling plans. *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete* **1** 70–81. MR 27.898, Zbl 118.34101

[1962b] Quality control for expensive items. Technical Report No. 57. Applied Mathematics and Statistics Laboratories, Stanford University, 29 pages.

Elfving's very first publication [1930] appeared well before his Dissertation [1-1934]. It is concerned with measuring depth on the open sea. There is no indication of why he became interested in this subject, but it may well be

that it qualified him for the 1935 Greenland expedition. Elfving provides deterministic arguments derived from hydrodynamics to correct measurements for salt content, temperature, pressure, location, and season. He tabulates the corrective percentages for the Gulf of Bothnia and the Gulf of Finland, for the months of May, August, and November.

The joint paper with Hustich [1944] provides an analysis of data on the radial growth of pine trees at the edge of a forest in northern Finland. The computations were carried out by student B. Qvist. It follows up a number of papers written by Hustich alone. Section II of the paper is authored by Elfving. He builds a statistical model to correlate growth with time and climate. Ilmari Hustich became Professor of Economic Geography and later Rector of the Swedish School of Economics and Business Administration, Helsinki. Bertil Qvist became Professor of Applied Mathematics of Åbo Akademi and later Rector of Handelshögskolan of Åbo Akademi.

Elfving and Whitlock [1950] evidently collaborated when Elfving visited Cornell. Whitlock was studying erythrocytes in sheep blood and wished to detect when their sustained growth became dangerous to the animal. As the authors say,

> "Regression technics would solve the problem, but they are too laborious to be worthwhile. ... Under these circumstances, a rank method was adopted."

That method had been suggested to Whitlock by William Feller who was in the Mathematics Department at Cornell. It involves the ranking of the time series of measurements from an individual sheep using a sum of Kendall's tau statistics, weighted across the blocks. For several individuals, the data were pooled and a normal approximation was used. John Tukey at Princeton provided improvements in the computing scheme. The method was still in use almost forty years later, see Taylor (1987).

Elfving's contribution [1957] to the Tufted Duck study of von Haartman is relegated to an Appendix rather than coauthorship. This is not surprising since von Haartman had been studying the Tufted Duck population West of Åbo for twelve years! It is made specifically clear that Elfving was involved at von Haartman's request. The three page Appendix

> "... is an attempt to develop a method for the analysis of incomplete material, i.e. such where the survey areas have not all been investigated during the same years."

A simple time series approach is used, and the probable cause of the variation in population is linked to severe winters in the periods 1939–42 and 1946–47.

The paper [1962a] on average outgoing quality level (AOQL) takes up work started by Lieberman and Solomon on multi-level continuous sampling plans. The results extend work by Derman, Johns, Lieberman, Littauer and Solomon. Results were provided for both block sampling and probability sampling. The paper [1962b] exists only as an unpublished technical report. It ends with integral equations for "expected loss" and for "duration" which require extensive computation for an approximate solution. Elfving concludes that minimizing the ratio of these two quantities remains a "forbiddingly complex problem. It seems that, in practice, hardly more than a comparison of a few tentative inspection rules can be attempted."

7. Decision Theory and Game Theory

[1952] Sufficiency and completeness in decision function theory. *Annales Academiæ Scientiarum Fennicæ, Series A I, Mathematica–Physica* **135**, 9 pages. MR 14.998c, Zbl 048.11903

[1962] Two-person Markov games. Technical Report No. 77. Applied Mathematics and Statistics Laboratories, Stanford University, 40 pages.

[1977] Peliteoria. In *Sovellettu matematiikka: 10 luentoa eri tutkimusaloilta*. Suomen aktuaariyhdistys, Espoo, 21–44.

A. Wald's book on *Statistical Decision Functions* appeared in 1950. The paper [1952] proves that Elfving was up to date with the current developments of his field. He summarizes the paper as follows:

> "The term sufficient statistics, introduced by R.A. Fisher, suggests that the statistic in question be *for any purpose at least as good as any other*. In this paper we give a formalization, in terms of decision functions, of the statement in italics, and show that the relation thus defined is actually equivalent with sufficiency. We restrict ourselves to the case that both distribution space, sample space, and decision space are finite, and to non-sequential procedures."

The paper [1962] exists only in the form of a technical report, the work on which was begun while Elfving visited Stanford in 1960, and was completed during a stay at the Institute of Mathematics, University of Århus, in the spring of 1962. In Århus, Elfving also gave a series of lectures on non-zero-sum games, probably based in part on the material in the report, and on the foundations of statistics, based in part on L.J. Savage's book *The Foundations of Statistics*.

In [1962] an attempt is made to overcome some of the well-known inherent difficulties associated with non-zero-sum games, such as the non-

existence of a unique "solution" of the game. Elfving does this by extending the notion of game by

> "... introducing temporal repetition, preplay negotiations, informations about the psychology of the players, or the like."

The main features of such a supergame are (a) infinite temporal iteration of the original game; (b) the payoffs of the supergame are the average payoffs of the single plays; and (c) the decisions of each player at any move are determined (possibly through randomization) by a fixed number of preceding moves. In view of (c), Elfving refers to both the strategies and the supergame as being Markovian.

The notions and results are given sociological interpretations, and a notion of social equilibrium is discussed, testifying to Elfving's continuing interest in placing mathematical-probabilistic theories in a broader context whenever possible. This applies, in particular, to decision theory, game theory and information theory, on which topics Elfving wrote several non-specialist articles with the above goal in mind. In [10-1973] he returns to the Markov game model.

The connections between game theory and optimum design of experiments are considered in [4-1963]. The paper [1977] is the published version of a lecture on game theory, delivered by Elfving to actuaries in Finland.

8. Bayesian Statistics

[1963] Beslut under ovisshet. *Arkhimedes* **15**(2) 19–26. Also *Nordisk Matematisk Tidskrift* **15** (1967) 79–89. MR 29.709, 37.1017, Zbl 158.17203

[1966] Robustness of Bayes decisions against the choice of prior. Technical Report No. 122. Department of Statistics, Stanford University, 28 pages.

[1968] Bayes statistics. Part I, II. *Skandinavisk Aktuarietidskrift* **51** 134–144, 145–155. Zbl 184.21905

[1978] Contribution to discussion of D.V. Lindley's paper "The Bayesian approach". *Scandinavian Journal of Statistics* **5** 18–19.

[1980] Bayes- ja Empirical Bayes-päätöksenteko. In *Suomen aktuaariyhdistyksen luentosarja 1979: Tilastoaineiston analysointi*. Suomen Aktuaariyhdistys ja Insinööritieto Oy, Helsinki, 121–131.

The expository article [1963], discussing the general foundation of Bayes decision theory, combines broad coverage with lucid clarity in the presentation. Starting out from postulates about the preferences of the decision maker, brief sketches are given of the axiomatic derivation of utility and subjective probability.

Paper [1968] is an extended version of an invited lecture series on Bayes inference and subjective probability, which Elfving delivered at the 1965 Nordic Conference on Mathematical Statistics in Århus. Elfving also mentions the objections raised against the Bayesian theory, which leads him to the compromise:

> "From a purely mathematical point of view, any theory is, of course, of interest as soon as it is correct, consistent, and of reasonable complexity."

Elfving finds that "one of the most important tasks for the workers on Bayes theory will probably be to investigate the *robustness* of the Bayesian methods with respect to variations in the prior distribution and in the utility function". He mentions that his [1966] technical report is an attempt in this direction.

In his contribution to the discussion [1978] of Lindley's paper Elfving says:

> "I would like to add that I am myself very much in favor of Bayes analysis; still, I think we should give the non-Bayesians the benefit of doubt."

In view of this confession it is surprising that there are only a few articles by him that concentrate on the Bayes approach.

The paper [1980] is the published version of a lecture on Bayes and empirical Bayes decision making, which Elfving delivered in 1979 to actuaries in Finland. It is part of a series of lectures on a variety of statistical topics, given by academic statisticians and applied mathematicians as a follow-up of a similar series of lectures in 1976; see also [7-1977].

9. Textbooks and Lecture Notes

[1944] Teknillisen Korkeakoulun matematiikka I–II kurssin tehtäväkokoelma. Helsinki University of Technology, 56 pages.

[1946] Todennäköisyyslaskenta. University of Helsinki, 118 pages.

[1951] (With O. Lokki) Matematiikka I–II kurssin tehtäväkokoelma. Helsinki University of Technology, 147 pages.

[1956] *Todennäköisyyslaskenta.* Tiedekirjasto **28**. Otava, Helsinki, 213 pages. [Second and third edition in 1964 and 1966, respectively.]

[1970] Todennäköisyyslaskenta II. Limes, Helsinki, 82 pages.

[1973] Stokastiset prosessit 1–2. Limes, Helsinki, 126 pages.

[1978] (With P. Tuominen) Todennäköisyyslaskenta II. Limes, Helsinki, 252 pages.

The collection of mathematical problems [1944] formed a complement to a text on differential and integral calculus used in the courses Mathematics I and II at the Helsinki University of Technology. The collection was written specifically for the war-time conditions of self-study, shortened courses and small, continously changing audiences. It is a clear indication of Elfving's wartime teaching obligations, in addition to his duties as a geodesist in the coast artillery. The [1951] reference builds on [1944].

Although written at an intermediate level, the lecture notes [1946] place probability explicitly within the framework of general measure theory and Lebesgue integration on Euclidean spaces, with due reference to the monographs of Saks and de la Vallée Poussin! Interestingly, there is a also a reference to dittoed seminar notes by Lars Ahlfors et al. on the theory of functions of a real variable (spring term 1942) and by Elfving on the set-theoretic foundations of probability (fall term 1942).

Building on the earlier lecture notes [1946], the book [1956] is a carefully written intermediate-level text in probability, almost half of which is actually devoted to topics from the theory of statistics. In the preface Elfving explicitly mentions the influence of Harald Cramér's two books *Sannolikhetskalkylen och några av dess användingar* and *Mathematical Methods of Statistics*. For many years, courses in probability at Finnish universities were based on [1956], which appeared in both a second and a third edition.

The lecture notes [1970] stem from a second course in probability, given by Elfving at the University of Helsinki in the 1960's. It is a compact presentation of the main concepts and results in measure-theoretic probability, additional details of which Elfving would fill in during his polished and well-organized lectures. Interestingly, the last chapter of [1970] contains an introduction to decision theory, reflecting Elfving's continuing interest in this area. Pekka Tuominen later expanded these notes into [1978], in order to make the material more suitable for self-study.

The lecture notes [1973] correspond to a course on stochastic processes, which Elfving gave at the University of Helsinki from the 1960's onwards.

Many Finnish probabilists and statisticians were first exposed to measure-theoretic probability and to stochastic processes by reading these lecture notes and by attending Elfving's lectures and seminars, which were a model of clarity of thought and exposition. Indeed, the importance of Elfving's work in promoting probability and statistics in Finland was recognized by the University of Helsinki through the founding of a professorial chair in this area in 1971.

10. Public Science

[1941] Sannolikhetskalkylens grunder i modern belysning. *Matemaattisten aineiden aikakauskirja* **1** 8–21.

[1942a] Om statistisk analys. In *Tekniska Läroverket i Helsingfors 25-års jubileumsskrift*. Liiketieto, Helsinki, 121–129.

[1942b] Förhållandet mellan läroverken och högskolorna i nuvarande undantagsläge. *Skola och hem* **5**(1) 34–39.

[1945a] Teknologens skolmatematik. *Matemaattisten aineiden aikakauskirja* **9** 41–49.

[1945b] Fundament och utvecklingslinjer inom sannolikhetskalkylen. *Matemaattisten aineiden aikakauskirja* **9** 141–150.

[1947a] Det matematiska sannolikhetsbegreppet. *Elementa* **30** 19–33. Zbl 029.36801

[1947b] Matematisk statistik – problem, metoder och tillämpningar. *Elementa* **30** 209–225. Zbl 029.22204

[1953a] Amerikanska universitetsförhållanden. *Arkhimedes* **5**(1) 28–31.

[1953b] Von Neumanns spelteori. *Arkhimedes* **5**(2) 17–21.

[1955] Språkets matematiska teori. In *Tieteen päivät Helsingissä 7.–9. tammikuuta 1954*. Suomalainen Tiedeakatemia, Helsinki, 234–241. Also in *Nya Argus* **47**(5), 1954, 61–64.

[1959] Spelteori och samhällsmönster. *Societas Scientiarum Fennica, Årsbok – Vuosikirja* **38**(B1) 1–14.

[1962] Spelteori och samhällsmodeller. *Ekonomiska Samfundets Tidskrift* **4** 226–238.

[1963] Peliteoria. In *Yhteiskuntatieteiden käsikirja I* (Eds. H. Waris et al.). Otava, Helsinki, 464–468.

[1965a] Information and esthetic evaluation. *Acta Philosophica Fennica* **18** 7–11.

[1965b] Matematikstudiet vid universiteten i Finland. *Nordisk Matematisk Tidskrift* **13** 123–126.

[1966] Information och estetisk värdering. In *Limes 1936–1966: 30-vuotisjulkaisu* (Eds. S. Kivelä et al.). Limes, Helsinki, 30–32.

[1968] Information och redundans. Societas Scientiarum Fennica, Årsbok – Vuosikirja **45**(B6) 1–14.

[1970] Research – experiences and view. In *Scientists at Work – Festschrift in Honour of Herman Wold* (Eds. T. Dalenius, G. Karlsson, S. Malmquist). Almqvist & Wiksell, Uppsala, 40–42.

[1971] Todennäköisyyden käsite. In *Logiikka ja matematiikka: Studia logica et mathematica*. Dilemma & Limes, Helsinki, 123–134.

[1973] A Markov game model for describing human behavior. In *Kutsu Filosofisen Tiedekunnan maisteri- ja tohtoripromootioon toukokuun 31 päivänä 1973*. University of Helsinki, 11–18.

[1975] *Årsfesttal i Åbo nation 1964–1975 och andra bidrag till Veritas et Jocus. Promotionstal vid Filosofiska fakultetens promotion 1973*. Karis Tryckeri, 24 pages.

Elfving wrote his first public science articles during World War II. While [1942b, 1945a] actually dealt with the specific didactic problems arising when students went from wartime gymnasia to wartime universities in Finland, the articles [1941, 1942a, 1945b] testify to Elfving's continued

scientific interest in probability and statistics during these years. The writing of articles directed to a wide non-specialist audience, as well as the delivery of such lectures, was to become an integral part of Elfving's scientific activities and, as pointed out by Mäkeläinen (1984, page 202) (translated from Finnish), "... was partly an acquired habit of working, which involved a certain effort to put new lines of thought into a general perspective". Indeed, this applied in particular to the new emerging fields of game theory and information theory, on which topics Elfving wrote several illuminating non-technical articles.

Among the articles on game theory [1953b, 1959, 1962, 1963, 1973], the first [1953b] is a succinct summary of the key ideas and results in the von Neumann–Morgenstern theory of games, indicating also the connections to Wald's theory of statistical decision functions. The articles [1959, 1962] are both based on general lectures given by Elfving at a session of the Finnish Society of Sciences and Letters and as part of a Studia Generalia-series of lectures at the University of Helsinki, respectively. They bear the typical marks of Elfving's non-technical writings: through a sequence of well-chosen examples from everyday life and the surrounding society, the reader is guided almost effortlessly and with a minimal use of formal machinery to the key concepts and ideas of a mathematically abstract theory. In 1963 Elfving even gave a radio-lecture on game theory and models of conflict, based on [1962].

In [1955, 1965a, 1966, 1968] Elfving considers the developments in information theory from several different perspectives. In [1955] he describes some of the linguistic as well as philosophical aspects of Claude Shannon's mathematical theory of communication and coding theory, again in terms of a series of well-chosen illustrative examples.

The article [1965a] was prepared for a Festschrift for Rolf Nevanlinna published by the Finnish Philosophical Society (*Acta Philosophica Fennica* **18**), and begins with a review of Shannon's fundamental ideas and results connecting the compression rate of a language (message flow) with its statistical structure, the optimal compression rate depending solely on the entropy of the language. As is well known, a low-entropy "stereotypic" language admits a higher compression rate than a "chaotic" language possessing high entropy, so that the most "chaotic" language is in a sense the most economical one. But Elfving goes off in another interesting and highly original direction. Namely, he points out that chaotic languages are not appealing and notes that

> "... we are ready to sacrifice part of the informatory efficiency of the language for the sake of other qualities which we call esthetic."

This leads him to ponder upon the proportions of random and regular components in esthetically pleasing pieces of art or other entities, and the roles of a certain amount of regular patterns and symmetry in an esthetically pleasing experience. He notes that

> "Our reflexions may be summed up to form what could be called a cybernetical view of esthetic value", and adds that "… the theories of information and stochastic processes provide models for a quantitative study of esthetic evaluation – if not of its more subtle aspects."

The article [1966] is essentially a version of [1965a] written in Swedish, but [1968] expands some of these ideas further.

As indicated by the latter quotation above, Elfving was also interested in the more technical aspects of the connections between information theory and patterns. Indeed, together with a graduate student, Pentti Suomela, he investigated the possibilities of using computer generated two-dimensional Markov random fields to produce random patterns exhibiting a certain symmetry without being deterministic. In the spring of 1968, Elfving actually gave a talk on Markov properties of two-dimensional stochastic fields at University College London, as part of the London Joint Statistics Seminar.

Among Elfving's other public science writings, the articles [1947a,b] are early reviews of developments in probability and statistics, respectively, while some of the other articles are on entirely different topics. Thus, in [1953a] Elfving describes the university system in the United States, drawing on his two-year experience at Cornell, and in [1965b] he describes, to a Nordic readership, the system of study of mathematics at Finnish universities. The collection [1975] includes Elfving's annual addresses to the Åbo Nation student body at the University of Helsinki, of which he was the "inspector" during the years 1964–1975. These entertaining and humorous addresses were cherished by the students, and reveal once again Elfving's broad cultural interests.

11. Book Reviews

[1953] Herman Wold in association with Lars Juréen: Demand Analysis. A Study in Econometrics. *Skandinavisk Aktuarietidskrift* **36** 103–105.

[1954] J.L. Doob: Stochastic Processes; and A. Blanc-Lapierre et R. Fortet: Théorie des fonctions aléatoires. *Mathematica Scandinavica* **2**(1) 165–167.

[1963] Samuel S. Wilks: Mathematical Statistics. *Statistisk Tidskrift* **1**(2) 177–178.

[1966] Selected Papers of Richard von Mises, Vol. 2. Probability and Statistics, General. Edited by G. Birkhoff. *Review of the International Statistical Institute* **34** 280–281.

[1969] A Selection of Early Statistical Papers of J. Neyman. Issued by "Students of J. Neyman at Berkeley". *Review of the International Statistical Institute* **37** 112–113.

[1971] Thomas Polfeldt: Asymptotic Results in Non-Regular Estimation. *Statistisk Tidskrift* **9**(1) 80–81.

[1972] Georg K. Chacko: Applied Statistics in Decision-Making. *International Statistical Review* **40**(3) 393.

[1977] D.B. Owen (ed.): On the History of Statistics and Probability. *Statistisk Tidskrift* **15**(1) 89–90.

[1978a] M. Kendall and R.L. Plackett (eds.): Studies in the History of Statistics and Probability, Vol. II. *Statistisk Tidskrift* **16**(3) 244–245.

[1978b] L. Gårding: Encounter with Mathematics. *Elementa* **61**(2) 100–101.

The book reviews by Elfving are evidently not written hastily, but exhibit the same clarity and thoughtful formulations that are characteristic of his writings in general. Several reviews also reveal his broad perspective of statistics and probability.

Reviewing Wilks' mathematical statistics book in [1963], Elfving notes the very brief coverage of the theory of experimental design, and expresses surprise at the heavy use of characteristic functions in lieu of ordinary matrix and vector notation and readily available matrix results, when deriving results in the parametric theory of estimation and hypothesis testing. Also, he notes that the brief chapter on decision functions is written "with little love" (translated from Swedish), and points out that this topic has, up to 1963, not been properly incorporated into the part of classical mathematical statistics which has found relevance in applications. Elfving goes on to make the prediction that (translation from Swedish) "… this [incorporation] will probably take place when the presently [in 1963] resurfaced Bayesian way of thinking has become dominant in statistical thinking."

In the review [1966] of the second volume of the selected papers of Richard von Mises, Elfving gives a clear and concise presentation of von Mises' fundamental notion of *Kollektiv*, putting it in the proper technical as well as historical perspective, and also predicts quite correctly that von Mises' differential calculus for statistical functionals "… may still have much to offer."

Elfving finishes the review [1978b] of Lars Gårding's *Encounter with Mathematics* with the following paragraph (translated from Swedish):

> "I have occasionally asked myself: If I were to be put in jail for some reason and were allowed to take with me *one* book, which one would I choose? Gårding's book is not a bad candidate."

12. History of Mathematics and Statistics

[1948] Några drag ur sannolikhetskalkylens historia. *Finsk Tidskrift* **4** 186–198.

[1967] Markov, Andrei Andreevich. In *Encyclopedia Britannica*. William Benton, Chicago and London, **14** 913.

[1977] Frithiof Nevanlinna in memoriam. *Arkhimedes* **29**(2) 104–106. Also in *Academia Scientiarum Fennica, Vuosikirja – Year Book 1977* (Ed. L. A. Vuorela). Suomalainen Tiedeakatemia, Helsinki, 1979, 77–79. MR 57.12115

[1980a] Rolf Nevanlinna. *Societas Scientiarum Fennica, Årsbok – Vuosikirja* **58**(C2) 1–4.

[1980b] Hjalmar Mellin. *Nordisk Matematisk Tidskrift* **28** 89–94. MR 82b.01043, Zbl 437.01027

[1981a] *The History of Mathematics in Finland 1828–1918*. Societas Scientiarum Fennica, Helsinki, 195 pages. MR 84c.01032, Zbl 476.01003

[1981b] Glimtar ur matematikens historia i Finland 1828–1918. *Societas Scientiarum Fennica, Årsbok – Vuosikirja* **59**(B2) 61–85. Translated into Finnish in *Katsauksia matematiikan historiaan* (Ed. J. Oikkonen). Gaudeamus, Helsinki, 1982, 144–173.

[1981c] J.W. Lindeberg. *Nordisk Matematisk Tidskrift* **29** 149–152. MR 83j:01055

[1982] Ernst Lindelöf. *Nordisk Matematisk Tidskrift* **30** 149–153. MR 84i:01084

[1983] Matematiikka Turun yliopistossa 1640–1713 ja algebran tulo Suomeen. In *Collegium Scientiae: Suomen oppihistorian kehityslinjoja keskiajalta Turun akatemian alkuaikoihin* (Ed. J. Nuorteva). Suomen kirkkohistoriallinen seura, Helsinki, 157–189.

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There are clear indications of Elfving's long-term interest in historical developments in probability and statistics, and in mathematics more generally; see the articles [1948] and [1967], and the book reviews [11-1966] and [11-1969]. Indeed, throughout his career, Elfving was not only concerned with the most recent technical results, but sought to place new developments (such as game theory, decision theory and information theory) into a broad context.

After he retired, Elfving turned to full-time research in the history of mathematics and statistics. The Finnish Society of Sciences and Letters entrusted him with the task of authoring the history of mathematics in Finland from 1828 until the year 1918 of Finland's independence from Russia. The choice of the Society could not have been better. The beautifully written and meticulously researched monograph [1981a] describes not only the historical developments of mathematics in Finland during that period, but embeds these developments seamlessly into the history of learning and culture in Finland, in keeping with Elfving's own broad cultural interests. The publications [1980b,

1981b,c, 1982] describe briefly some notable personalities selected from the 56 mathematicians featured in [1981a].

In the study [1983], Elfving describes the mathematics of the seventeenth century in Finland. He discusses the early background of European algebra, going back to the influence of Babylonian algebra on early Greek mathematics. Indeed, this appears to have been another long-standing interest of his, as he had given a series of lectures on the topic in the fall of 1955, see Lehti (1984).

When the work on [1981a] was completed, the Finnish Society of Sciences and Letters asked Elfving to prepare a historical account of the third half-century 1938–1987 of the Society. Having served the Society in various capacities for decades, Elfving was singularly qualified for this task. Interestingly, the previous account, covering the first century of the Society, had been written by his father. Elfving had completed the work at the time of his death, except for the final chapter on developments from 1980 onwards which was written by Gösta Mickwitz. The joint work appeared as the book [1988].

The article [1985] is based on an invited presentation that Elfving delivered at the First Tampere meeting in 1983. It is a delightful read, ending in the following excerpt, which shows how Elfving felt about scientific discoveries and learning at the end of a long and productive life:

> "After such a little lookback into the history of learning, one may ask what insights we may gain from it.
>
> First of all: The same truth may be discovered in several places, and will usually be attributed, not necessarily to the author who has first found it, but to one that belongs to a school which is prepared to listen to him.
>
> And second: Lots and lots of achievements will, often rightly but sometimes wrongly, never reach the text-book heaven of established truth; they will descend to the underworld of the university libraries — to be perhaps one day rediscovered by some student of learning.
>
> But all this one should not be too sad about. It is the law of life. The forgotten authors may still have given impulses to pupils, and enriched their thinking. And the joy of understanding that they have experienced adds to the total of happiness that makes life worth while."

Acknowledgements

We are very grateful to Gustav Elfving's sons, Jörn and Tord, for extremely helpful discussions about biographical details, and to Anders Ekholm, Johan Fellman, Søren Johansen and William J. Studden, for valuable comments on an earlier draft. N.R. Draper and F. Pukelsheim acknowledge with thanks support by the Alexander von Humboldt-Stiftung, Bonn, through a Max-Planck-Award for cooperative research. The work of K. Nordström has been supported by a grant from the Ella and Georg Ehrnrooth Foundation.

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Statistics, Registries, and Science

Experiences from Finland

Edited by
Juha Alho

Published by
Statistics Finland
1999

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