

Chaos and Fractals

Exercise Set 9

Due: Monday, June 30, 2003, at 16.00 hours in the mailbox “Chaos and Fractals”.

Exercise 33

Prove that a metric space is compact if and only if it is complete and totally bounded.

Exercise 34

On the unit square $[0, 1]^2$ consider the iterated function system given by the three mappings $w_1(x) = Ax$, $w_2(x) = Ax + \begin{pmatrix} 1/2 \\ 0 \end{pmatrix}$, $w_3(x) = Bx + \begin{pmatrix} 1 \\ 1/2 \end{pmatrix}$ where

$$A = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & -1/2 \\ 1/2 & 0 \end{pmatrix}.$$

Get an idea (by hand or a computer) of what the attractor of this IFS looks like.

Exercise 35

Prove that the attractor of an iterated function system consisting of two affine mappings $w_1(x) = ax + b$, $w_2(x) = cx + d$ on \mathbb{R} is either connected or totally disconnected.

Hint: Distinguish whether $|a| + |c| < 1$ or ≥ 1 and use the Lebesgue measure on \mathbb{R} .

Exercise 36

Using the Collage Theorem, find an iterated function system whose attractor looks like the fractal on the back of this sheet.

Write a computer program in order to verify your result.