

Chaos and Fractals

Exercise Set 10

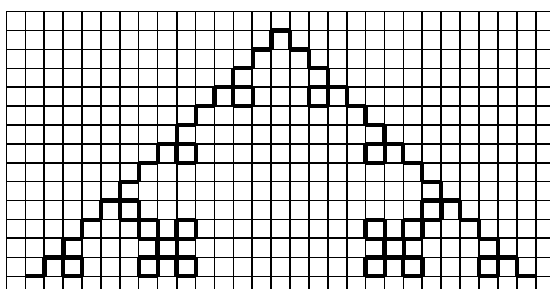
Due: Monday, July 7, 2003, at 16.00 hours in the mailbox “Chaos and Fractals”.

Exercise 37

Given a complete metric space (X, d) and the corresponding space of fractals $(\mathcal{H}(X), h)$, prove that for any $M \in \mathcal{H}(X)$ and $\varepsilon > 0$ there is an iterated function system $\{w_1, \dots, w_n\}$ whose attractor A_w satisfies $h(M, A_w) < \varepsilon$.

Exercise 38

Find an iterated function system whose third iterate of the corresponding Hutchinson operator w applied to the set $I := \{(x, 0) \in \mathbb{R}^2 : 0 \leq x \leq 1\}$ looks as follows:



Prove that the length of the curve $w^k(I)$ tends to ∞ as $k \rightarrow \infty$.

Exercise 39

Determine an iterated function system whose attractor looks like the picture on the back of this sheet.

Exercise 40

A point a on the attractor A_w of an iterated function system $\{w_1, \dots, w_n\}$ on a metric space (X, d) is called **periodic** if there exist finitely many numbers $i_1, \dots, i_p \in \{1, \dots, n\}$ such that $a = w_{i_1} \circ \dots \circ w_{i_p}(a)$.

Prove that A_w is the closure of its periodic points.